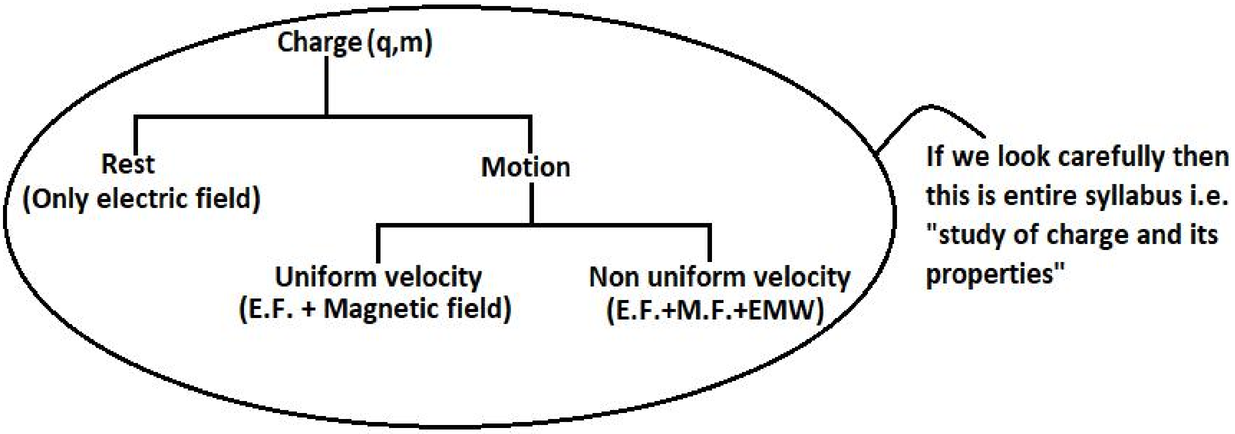
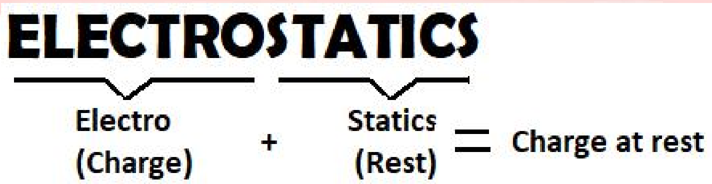




Electrostatics is study of charge & its properties under rest condition.

Note: In electrostatics motion of charge for very small time is allowed.

Electric charge and Coulomb's law

Electric charge: - It is intrinsic property of fundamental particles of matter due to which electromagnetic force is present in nature.

Note: - Electron, proton and neutron are fundamental particles of matter. These are made up by "quarks".

Charge on up quark is $+\frac{2}{3}e$

Charge on down quark is $-\frac{1}{3}e$

1) Electrons and protons are made up by "up quarks" and "down quarks", so that total 3 quarks are used to form each of them. Find the possible combinations of them?

Facts about charge: -

- Without mass existence of charge is not possible.
- Smallest possible charge in nature is $(1.6 \times 10^{-19} C = e)$. This is called quanta or quantum of charge also.

- Charge is scalar quantity. Its SI unit is coulomb (C).

Properties of Charge: -

- **Like charges repel while unlike charges attract.**

2) If two charges q_1 and q_2 repel each other then identify the correct option/options.

a) $q_1q_2 > 0$ **b)** $q_1 + q_2 > 0$ **c)** $\frac{q_1}{q_2} > 0$ **d) all**

3) Which is a true test of electrification of a body, attraction or repulsion?

Ans. Repulsion

- **Electric charge follows conservation law (Conservation of charge)** $\Rightarrow$ It means net charge of any isolated system remains always conserved.

Note: *In nature energy + mass, linear momentum, angular momentum and electric charge are the physical quantities which are always conserved for any isolated system*

4) Mark out the correct options.

(a)The net charge of the universe is constant

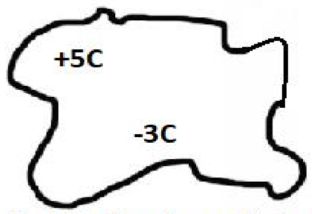
(b)The net positive charge of the universe is constant

(c)The net negative charge of the universe is constant

(d)The total number of charged particles in the universe is constant

Ans: a

- **Electric charge follows additivity rule (Additivity of charge)** $\Rightarrow$ It means net charge of any isolated system is obtained by the algebraic addition of individual charges present in the system.



Isolated system of charge

$q_{net} = (+5) + (-3) = +2C$

- **Electric charge follows law of quantization (Quantization of charge)** $\Rightarrow$ It means net charge of any body is made up by the integral multiple of electronic charge $(1.6 \times 10^{-19} C)$.

  $q = \pm ne$

  Here, n = 1, 2, 3, 4,……∞ (number of electrons used to form the charge)

  $n \neq 1/2, 3/5, 7/12 \; ....$

Note: *Quantum of charge is electronic charge.*

5) **"Charge of a body can't be zero" this statement is true or falls?**

   Sol. True

6) **Is it possible that net charge on a body is 2.5C?**

   Sol. Yes

7) **Can** $0.8\times10^{-19}C$ **of charge be given to a conductor?**

8) **Find no. of electrons used to form 1 C charge.**

Sol. $q = ne$

$n = q / e$

$n = \frac{1}{1.6 \times 10^{-19}}$

$n = 625 \times 10^{16}$ (This is a very large number)

9) **If net charge on an object is 2.5C then find number of electrons transferred from it.**

10) **While dealing with very large amount of charge (macro charge) we can ignore quantization of charge why?**

Sol. Because quantum of charge (smallest possible value of charge) is electronic charge $e = 1.6\times10^{-9}C$ which is very small. Hence, we can ignore quantization in case of **macro-charge.**

11) **How much of megacoulombs positive charge is present in 2 mol of neutral hydrogen atoms?**

12) **The charge on a proton is $+1.6\times10^{-19}C$ and that on an electron is $-1.6\times10^{-19}C$. Does it mean that the electron has a charge $3.2\times10^{-19}C$ less than the charge of a proton?**

Ans: No

13) **Is there any transfer of mass when electrons are transferred from one substance to another?**

- **Invariance of charge:** Net charge of a particle is independent from the speed of the particle, otherwise we can't explain neutrality of an atom. [Mass depends on speed as following equation]…..

  $$m = \frac{mo}{\sqrt{1-\frac{v^2}{c^2}}}$$

  $m = mass\ under\ motion$, m_0 = mass at rest, v = speed of particle, c = speed of light

  $\Rightarrow If\ v\uparrow, then\ m\uparrow$

  From above equation it is also clear that speed of a particle can not be equal to speed of light otherwise mass of that particle becomes infinite, which is not possible.

- **Charge at rest produces electric field only.**
- **Charge moving with constant velocity produces EF & MF both.**
- **Charge moving with some acceleration produces EF, MF and EMW also.**

Methods of charging of a body:

a) Charging by friction

b) Charging by conduction or charging by contact

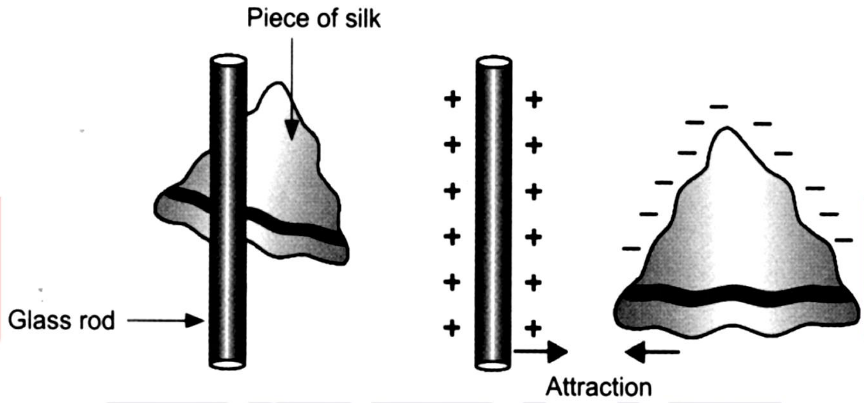
c) Charging by induction

a) **Charging by friction: -**

i) It is Fundamental method of charging of a body.

ii) In this method both bodes acquire equal and opposite charges.

iii) After charging both objects attract each other.



Note: -

- **Work function of a metal (W):** The minimum energy required to eject an electron from a metal surface is known as its work function. S.I. Unit = J
- During charging by friction a body having low work function becomes positively charge and vice versa.

14) If a body is charged by rubbing it, its weight:

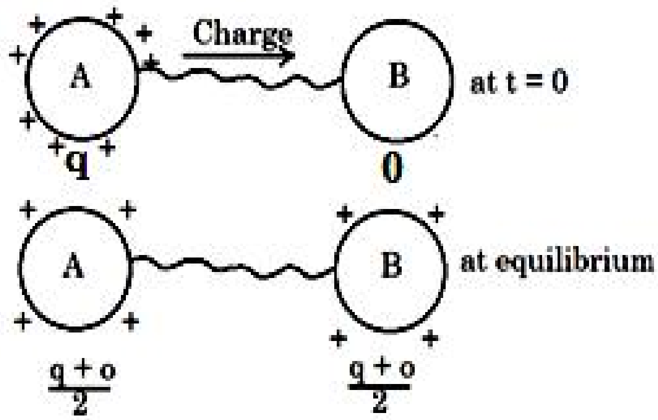
(a)remains precisely constant (b)increases slightly

(c)decreases slightly (d)may increase slightly or decrease slightly

15) Why does a phonograph-record attract dust particles just after it is cleaned?

16) If we rub two objects A and B of which work functions are + 5eV and +10eV and electrons transferred from one two other are 25×10^{18} then find net charge on A and B after the process.

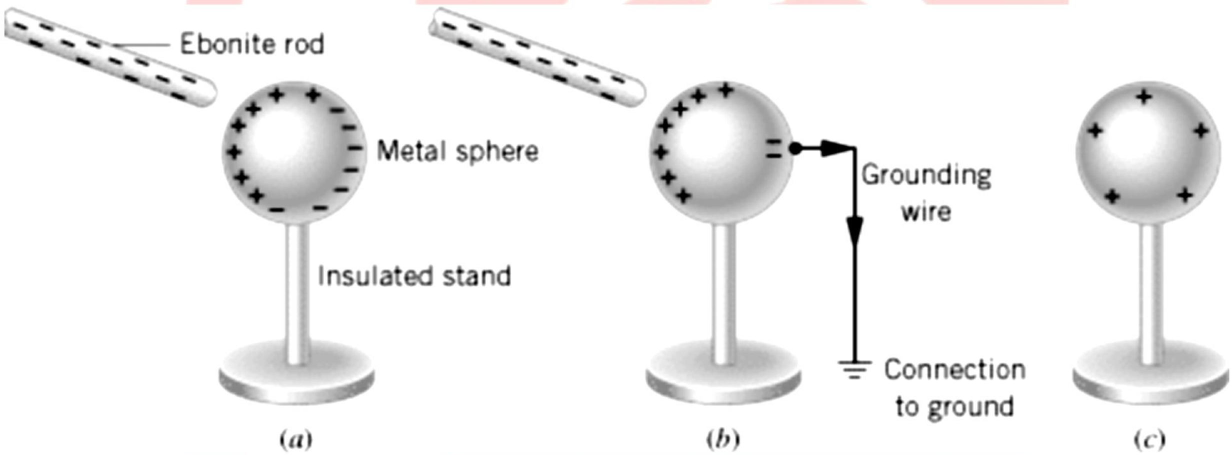
b) Charging by Conduction or Charging by Contact:-



When we brought charged identical bodies in contact then **net charge** of the system **equally divides** on each of the bodies.

c) Charging by Induction:

Method-I

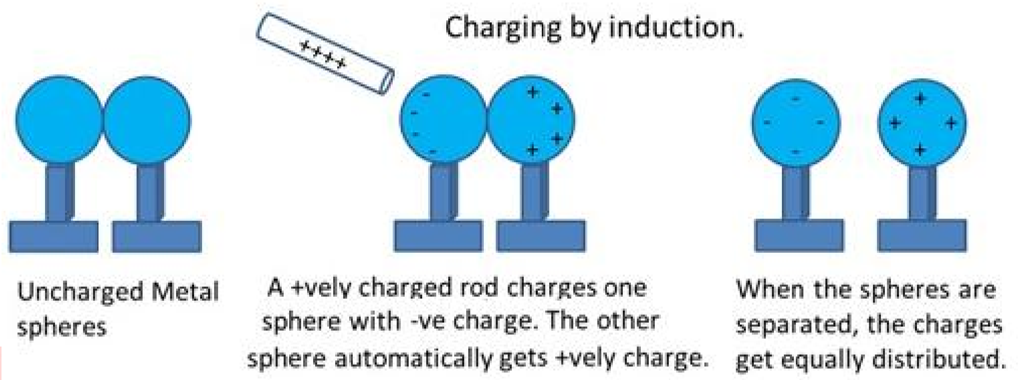


Note:

- Earth is an ocean of electrons.
- Earth can accept or donate infinite electrons.
- After earthling of a body its potential becomes necessarily zero but charge may or may not be zero [We will discuss about electric potential in next chapter]
- Positive charge always flows from high potential to low potential

17) A person standing on an insulating stool touches a charged insulated conductor. Will the conductor get completely discharged?

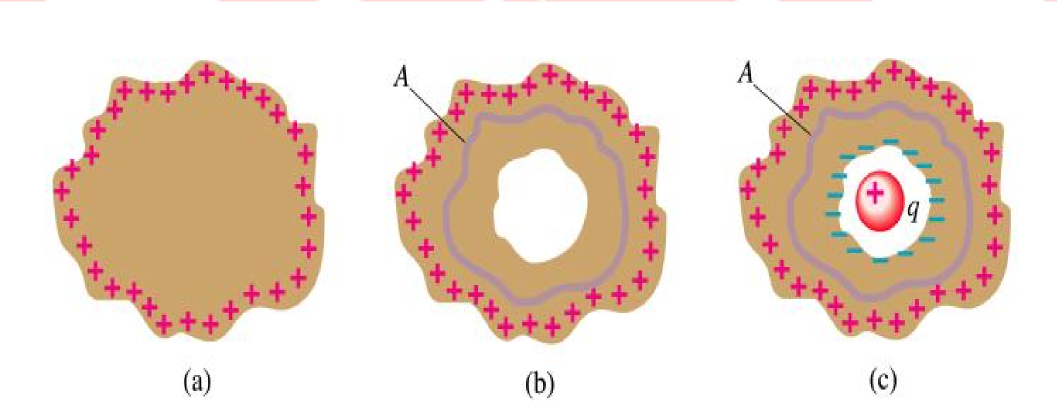
Method-II



i) It is a method of charging in which we can charge a body without touching it.

ii) During induction, on opposite faces equal and opposite charges appear [In case of induction between conductors otherwise induce charge is given by $Q\left(1-\frac{1}{K}\right)$.

An important situation of charging by induction:



18) When a charged comb is brought near a small piece of paper, it attracts the piece. Does the paper become charged when the comb is brought near it?

Ans: No

19) Can two like charged bodies attract each other?

Ans: Yes

20) Can two like point charges attract each other?

Ans. No

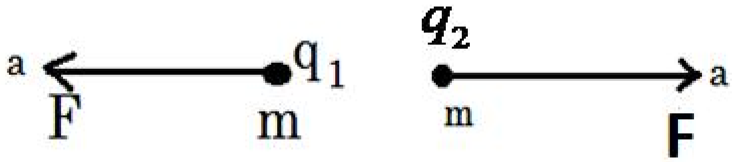
21) Can a charged body attract an uncharged body?

22) A small metallic hallow sphere is charged by +10 C. If we inject a point charge -2C at its center then find out net charge on the inner surface and outer surface of the sphere.

Ans. $q_{inner} = +2C$
$q_{outer} = +8C$

Coulomb's Law:- According to coulombs law if two **point charges** are separated by some distance then they exert equal and opposite forces on each other.

Magnitude of the force is directly proportional to the product of magnitude of both charges & inversely proportional to the square of distance between them.



$$F \propto \frac{|q_1||q_2|}{r^2}$$
$$F = k\frac{|q_1||q_2|}{r^2}$$

Here k is a proportionality constant (Coulombian force constant) its value is given by:

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \text{ X } 10^9 \frac{Nm^2}{c^2} \quad \text{(for air)}$$

ε_0 is called electrical permittivity of free space, $\varepsilon_0 = 8.85\times10^{-12}C^2N^{-1}m^{-2}$.

In general (for other medium) $k = \frac{1}{4\pi\varepsilon}$, ε is permittivity of the medium, its value is different for different type of medium.

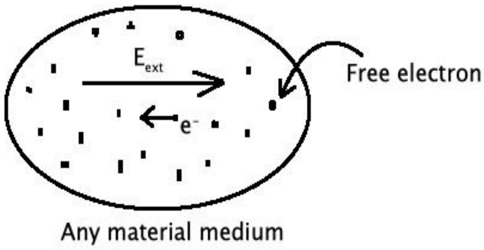
Electrical Polarization of a medium: When an electric field is passed through a medium then free electrons of that medium experience electric force in opposite direction of electric field and form induced electric field this phenomenon is called electrical polarization of the medium.

- Induce electric field always opposes to external electric field.

- Inside a material always a net electric field is present which is given by the vector addition of external electric field and induced electric field.

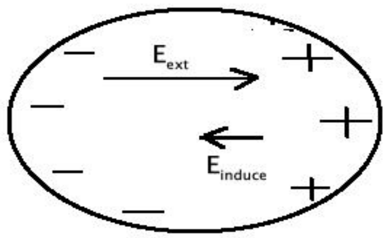
$$\vec{E}_{net} = \vec{E}_{ext} + \vec{E}_{induce}$$

$$E_{net} = E_{ext} - E_{induce}$$



Dielectrics: Those materials which have limited number of free electrons are called dielectrics.

- Different types of dielectrics have different number of free electrons hence their electrical response is also different.
- Inside a dielectric net electric field can be present which is always less than external electric field. [due to opposition of induce electric field]



$$\vec{E}_{net} = \vec{E}_{ext} + \vec{E}_{induce}$$

$$E_{net} = E_{ext} - E_{induce}$$

$$E_{net} \neq 0$$

- Electrical opposition of a dielectric is measured a physical quantity which is called "dielectric constant (K)" of the medium. Its value depends on the number of free electrons per unit volume inside the medium.

- Higher the value of K the medium provides more obstruction to external electric field as well as electric force.
- It means on increasing the value of K **net** electric force between two charges decreases.
- Note that on changing medium between two charges electric force between the charges remains same [this is superposition law], while net electric force changes.

Electrical permittivity of a medium (ε): It is the measurement of degree of polarization of a medium, when an electric field is passing through it.

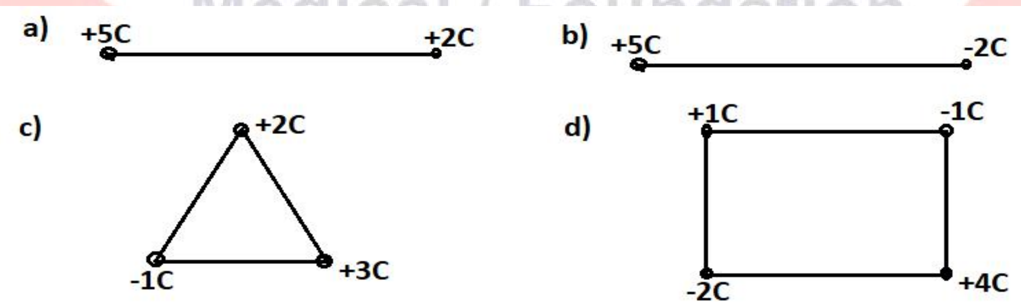
If polarization of a medium is high then force between any two charges in that medium is low. Value of ε for a medium is measured by the theory of electro-magnetism.

$c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$, c is the speed of light in air

$v = \dfrac{1}{\sqrt{\mu \varepsilon}}$, v is the speed of light in other medium

Note: Coulomb's law is applicable up to atomic level also.

23) Represent direction of all the forces acting on each charge in the given diagrams.



24) Find out magnitude and directions of net force acting on each charge, distance between them is 10m.

25) Is there any lower limit to the electric force between two particles placed at a separation of 1 cm?

Ans: Yes

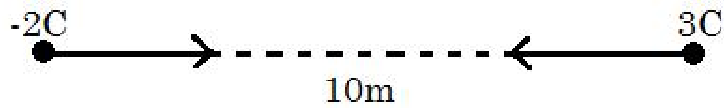
26) Find minimum possible force between two charged particles placed at a distance of 1m.

27) Two equally charged particles A and B are placed at some distance. The particle A is slightly displaced towards B. Does the force on B increases as soon as the particle A is displaced? Does the force on particle A increases as soon as it is displaced?

Ans: No, Yes

28) Find force between two charges -2C and 3C placed at a distance of 10m in air.

Sol.



$$F = \frac{9 \times 10^9 \times (2) \times (3)}{10^2}$$

$$F = 54 \times 10^7 \text{ W}$$

29) Prove that electric force is always greater than gravitational force.

Properties of Coulomb Force:

(i) It can be attractive and repulsive both.

(ii) It depends on medium.

(iii) It is central force: It means this force always acts along the line joining the centre of mass of both the charges.

(iv) It is conservative force: It means work done due to this force is independent from the path followed by charge particles

Or

Net work done by electric force in a round trip is always zero.

(v) It follows inverse square law.

$$F \propto \frac{1}{r^2}$$

Now for clear understanding of numerical lets revise vector addition.

Revision of Vector Addition:

We know that if $\vec{R} = \vec{A} + \vec{B}$ then there are two hidden meaning in this equation

1. $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$, here R is magnitude of $\vec{R}$

2. $\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$, here α is the angle of resultant with respect to direction of $\vec{A}$

I observe that in electrostatics approximately all the questions are based on the following conditions only, hence the students must have to remember results of these conditions directly to make fast calculations

(i) $If\ A = B\ \&\ \theta = 90° \ then\ R = \sqrt{2}\,A$

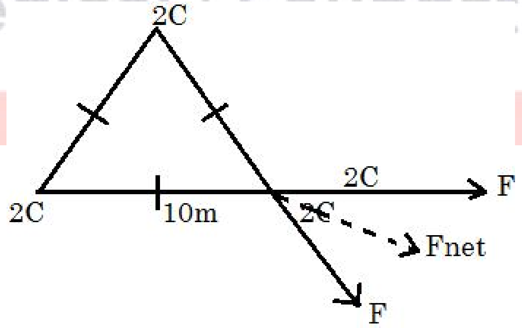
(ii) $If\ A = B\ \&\ \theta = 60° \ then\ R = \sqrt{3}\,A$

(iii) $If\ A = B\ \&\ \theta = 120° \ then\ R = A$

(iv) $If\ \theta = 0° \ then\ R = A + B$

(v) $If\ \theta = 180° \ then\ R = A - B$

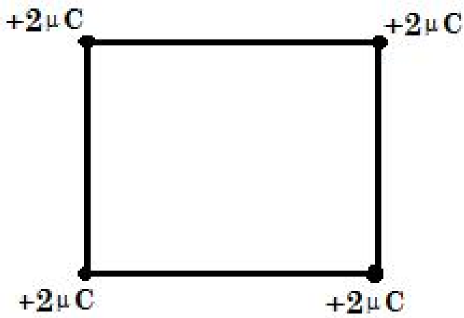
30) Find net force at any one of the charge.



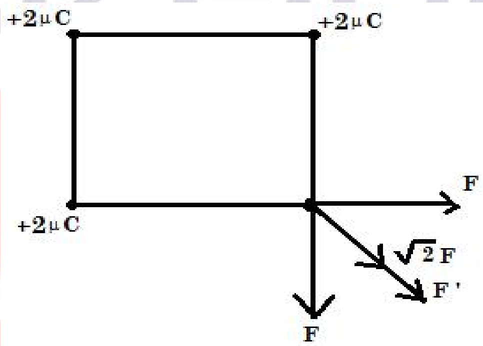
Sol. $F_{net} = \sqrt{3}\,F$

$\Rightarrow F_{net} = 36\sqrt{3}\ X\ 10^7\ N$

31) Find net force at any one of the charges placed at the corners of the given square of side 10 m.



Sol.:

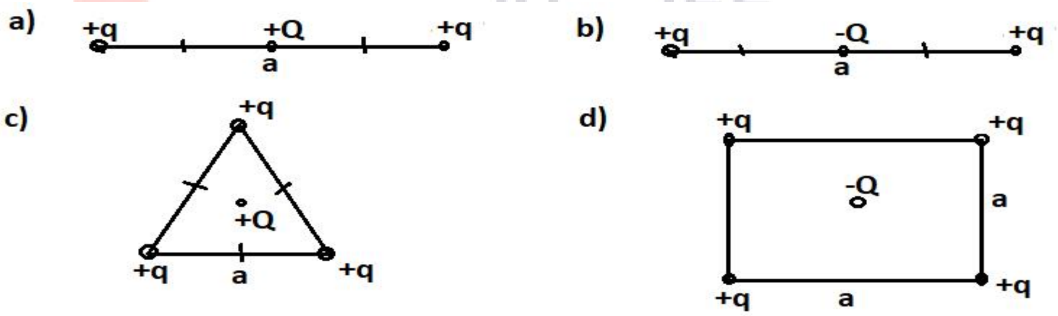


$$F_{net} = F' + \sqrt{2}F$$

$$= \frac{k2 X\ 2X\ 10^{-12}}{(10\sqrt{2})^2} + \sqrt{2}.\frac{k2 X\ 2X\ 10^{-12}}{(10)^2}$$

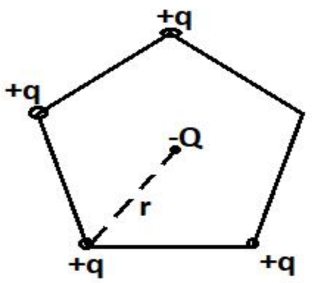
$$= \frac{k\ 4 X\ 10^{-12}}{100}\ [\frac{1}{2} + \sqrt{2}]\ = 4k\ X\ 10^{-14}\ [\frac{1}{2} + \sqrt{2}]\ N$$

32) Find net force at the central charges of the following given diagrams:



Conclusions: In a regular geometry if all the charges placed on the vertices are identical then for any type of charge placed at the centre net force is always zero.

33) Find net force at the central charge of the following given diagram:



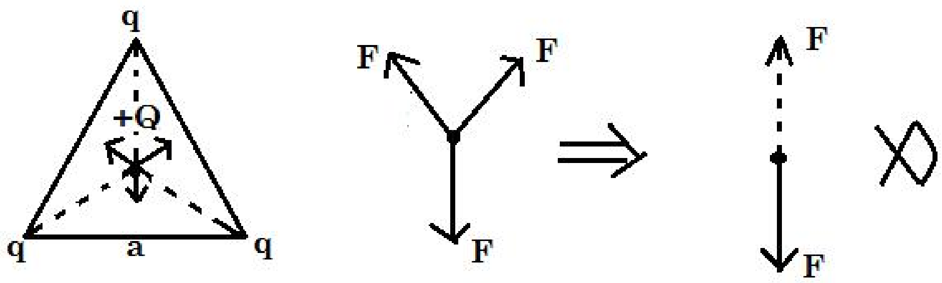
34) If three identical charges each of q are placed on the vertices of an equilateral triangle then find net force acting on another charge Q placed at the centre of the triangle.

Sol.

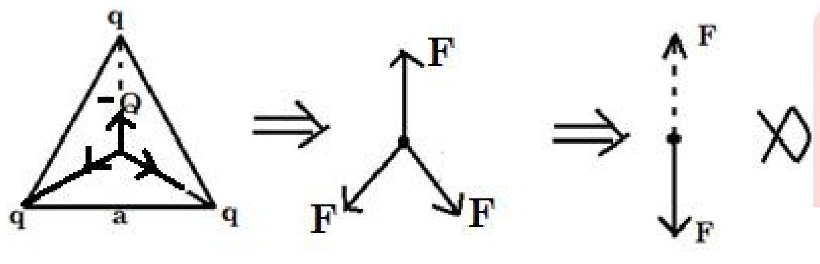
In a quadrilateral if the surrounding charges are identical then for centre charge to be in equilibrium we can take any value and any sign of the central charge.

Explanation:

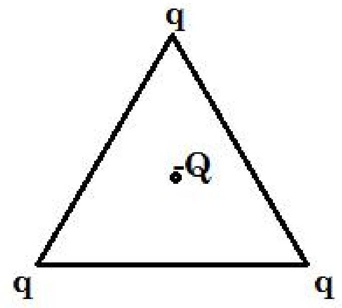
Case-1 For Positive sign of charge at centre:



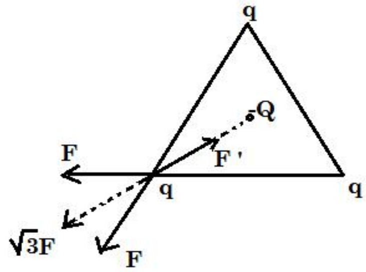
Case-2 for negative sign:



35) Find net force at any one charge placed at the vertex of an equilateral triangle as shown in figure.



Sol.

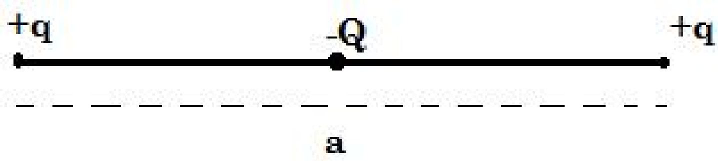


$$\sqrt{3}k\frac{q^2}{a^2}=\frac{kQ}{\left(\frac{a}{\sqrt{3}}\right)^2}$$

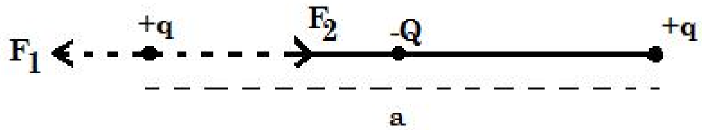
$$\frac{\sqrt{3}}{3}q=|Q|$$

$$Q=\frac{-q}{\sqrt{3}}$$

36) **Two like charges +q, +q are fixed at some distance a. Find out magnitude and sign of a third charge which must be placed at the centre such that the central charge will be in equilibrium.**



Sol.



$$F_1=F_2$$

$$\frac{q^2}{a^2} = \frac{q\,Q}{(\frac{a}{2})^2}$$

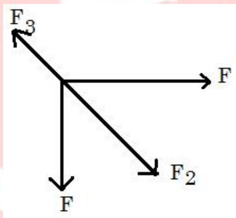
$$\frac{q}{4} = |Q|$$

$$Q = \frac{-q}{4}$$

Conclusion:- For the system to be in equilibrium surrounding charges and central charge must have opposite sign.

37) In above question find magnitude & nature of central charge which must be placed such that the system will be in equilibrium.

Sol.



$$F_2 + \sqrt{2}F = F_3$$

$$\frac{K\,q^2}{(\sqrt{2}a)^2} + \frac{\sqrt{2}\,K\,q^2}{(a)^2} = \frac{K\,q\,Q}{(\frac{\sqrt{2}}{2}a)^2}$$

$$\frac{q^2}{2a^2} + \frac{\sqrt{2}\,q^2}{a^2} = \frac{q\,Q}{\frac{2a^2}{4}}$$

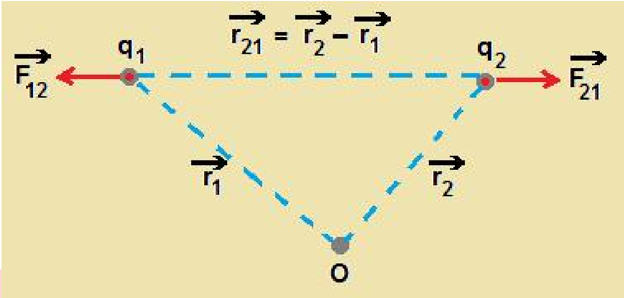
$$\frac{q}{2} + \sqrt{2}\,q = \frac{4\,Q}{2}$$

$$\frac{q + 2\sqrt{2}\,q}{2} = 2Q$$

$$|Q| = q\,[\frac{1}{4} + \frac{1}{\sqrt{2}}]$$

$$Q = -q\,[\frac{1}{4} + \frac{1}{\sqrt{2}}]$$

Vector form of Coulomb's law:- Let two charges q_1 and q_2 are such that their position vectors are $\vec{r}_1$ and $\vec{r}_2$ as shown:



Then force on q_2 due to q_1 is given by:

$$\overrightarrow{F_{21}} = \frac{k\, q_1 q_2}{r^2}\hat{r}_{21}$$

$$\overrightarrow{F_{21}} = \frac{k\, q_1 q_2}{r^3}\vec{r}_{21} \qquad \left[\hat{r} = \frac{\vec{r}}{r}\right]$$

- Here $\vec{r}$ is towards the charge at which we have to find the force.
- Charges must be placed with signs.
- Above equation can be also written as:

$$\vec{r}_1 + \vec{r} = \vec{r}_2$$

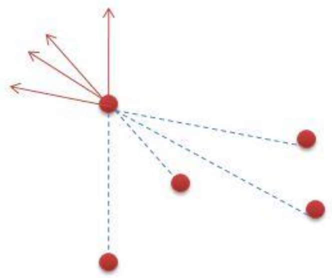
$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\vec{F}_{21} = \frac{k\, q_1 q_2}{\left|\vec{r}_2 - \vec{r}_1\right|^3}\left(\vec{r}_2 - \vec{r}_1\right)$$

Superposition law of Coulomb force:- According to superposition law net force acting on any charge is given by the vector addition of all the individual forces acting on that charge.

Or

Net force acting on a charge particle is independent from presence or absence of other charge particles.



$$\overrightarrow{F_{net}} = \overrightarrow{F_{12}} + \overrightarrow{F_{13}} + \overrightarrow{F_{14}} + \overrightarrow{F_{15}}$$

Revision SHM: In every type of stable equilibrium when we displace the particle slightly from its stable equilibrium position and release then the particle shows SHM.

21)Does the force on a charge due to another charge depend on the charges present nearby?

Ans: No

To find time period:

Step-1 we have to assume a very small displacement 'x' from mean position.

Step-2 we have to find net restoring force at this new position. In case of SHM that force must be a linear function of assumed displacement and opposite to it.

Step-3 Compare above restoring force with its general form i.e. $\vec{F} = -m\omega^2\vec{x}$ and find ω then T.

38) A particle of mass m and charge +q is located midway between two fixed charged particles each having a charge +q and at a distance 2L apart. Assuming that the middle charge moves along the line joining the fixed charges, calculate the frequency of oscillation when it is displaced slightly.

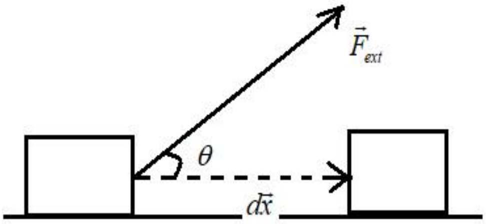
Sol. $F_{net} = kq^2 \left[\frac{4Lx}{(L^2-x^2)^2}\right]$ (x <<< L)

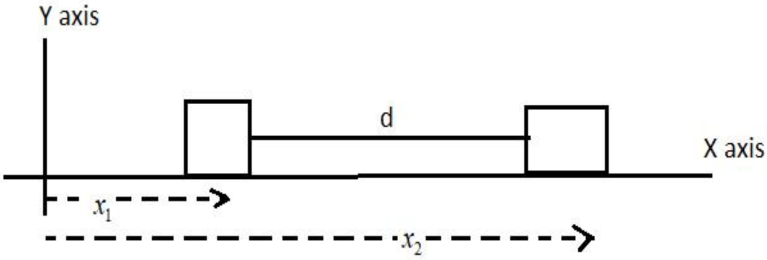
$$F_{net} = \frac{1}{4\pi\in_0} \cdot \frac{4q^2}{L^3}\, x$$

$$F_{net} = \frac{1}{\pi\in_0} \cdot \frac{q^2}{L^3}\, x$$

$$\omega = \sqrt{\frac{q^2}{\pi\in_0 m L^3}}, \quad \omega = \frac{q}{L}\sqrt{\frac{1}{\pi\in_0 m}}$$

Work done by a force:





$$w = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{x}$$

$$\boldsymbol{w = \int_{x_1}^{x_2} F\, dx \cos\theta}$$

If F is constant then,

$$w = \int_{x_1}^{x_2} F\, dx \cos\theta$$

$$W = F(x_2 - x_1)\cos\theta$$

$$W = Fd\cos\theta$$

Work energy theorem (WET):

$W_{net} = \Delta K$ (work done due to all the forces)

WET is applicable without any condition.

(i) Energy conservation law

$K_i + U_i = K_f + U_f$

To apply energy conservation condition is that work must be done by the conservative forces only.

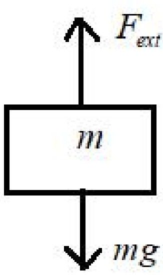
39) Find out force required to lift a mass m up to a height h without acceleration?

Sol.

$$F_{net} = ma$$

$$F_{ext} - mg = m(0)$$

$$F_{ext} = mg$$



Conclusion: To displace a block external force required must be equal to resistive force only.

40) Find out work done by external force to lift a mass m up to a height h without acceleration?

Sol. $W = mg(d)\cos 0$

$W = mgh$

41) Find out work done by gravity to lift a mass m up to a height h without acceleration?

Sol. $W = mg(d)\cos 180$

$W = -mgh$

42) Find out work done by external force to drop a mass m from a height h without acceleration?

Sol. $W = mg(d)\cos 180$

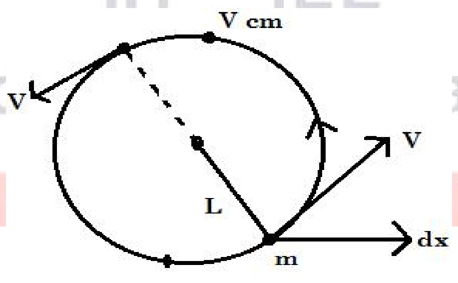
$W = -mgh$

43) Find out work done by gravity to drop a mass m from a height h without acceleration?

Sol. $W = mg(d)\cos 0$

$W = mgh$

44) In the given diag. during half rotation find out work done by the tension.



Sol.:- $W = \int \frac{mv^2}{L}(dx)\cos 90°$ or W = 0

Conclusion:

(i) During downward motion, work done by gravity (+mgh)

(ii) During upward motion, work done by gravity (-mgh)

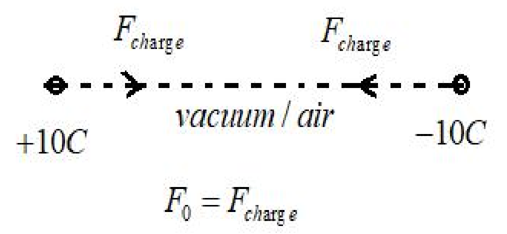
(iii) If force is perpendicular to instantaneous displacement then work done by the force is always zero.

(iv) In a conservative field to displace a block without acceleration $W_{ext} = -W_{\text{int}}$

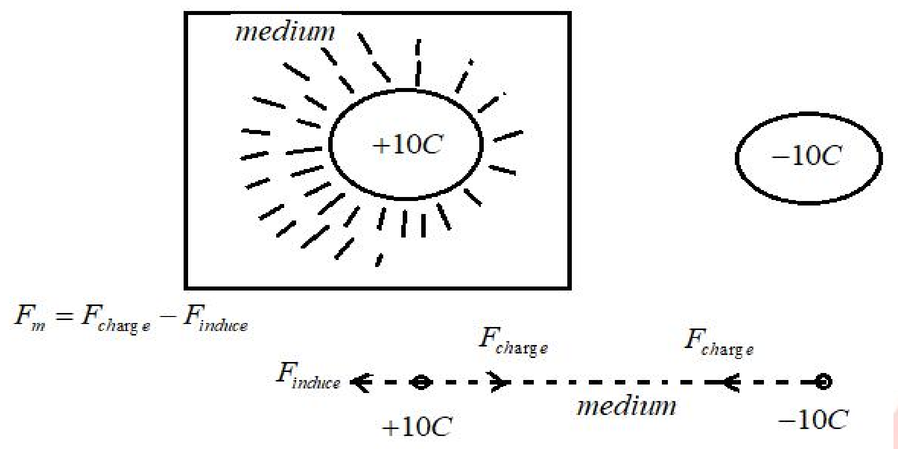
45) If we rotate a charge q in a circular path at the centre of which an other charge Q is placed then find out net work done by electric force in one complete rotation.

Effect of medium on coulomb force:

In air force on any charge is only due to other placed charge near it



In a medium force on any charge is net effect of force due to other placed charge and induced charge in the medium also.



Dielectric constant (K) of a medium or relative electrical permittivity (ε_r):

Maximum coulomb force between two charged particles is observed in air. If we introduce any other medium then net value of coulomb force decreases by a factor [it occurs due to dielectric polarization of the medium] i.e. dielectric constant of the medium and it is given by:

$$F_m = \frac{F_0}{K}$$

here K is dielectric constant of medium.

Definition of Dielectric constant of a medium (k): It is the ratio of electrical permittivity of a medium to permittivity of free space.

$$K = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$$

K = 1 for air

K>1 for other medium

K = ∞ for conductor [due to this reason net electric field inside a conductor is zero]

Note: Naturally there is no any material for which value of k<1 {without using external energy}

22)If we submerged a system of two charged particles in a liquid then the force between both charged particles:

(i) increases (ii) decreases (iii) remains same (iv) none of these

Sol. remains same

23) If we submerged a system of two charged particles in a liquid then the net force between both charged particles:

(i) increases (ii) decreases (iii) remains same (iv) none of these

Sol. decreases

24) Define dielectric constant of a medium in terms of electric force.

Sol. It is the ratio of electric force in air to the electric force in given medium.

Electric Field

Every charge produces its own electric field in its surrounding.

Strength of electric field at different points is different that is measured by electric field intensity $(\vec{E})$ at that point.

Electric field due to a point charge produced at its own place is always 0

Note: *Electric field is the characteristics of a point due to a charge placed in its surrounding.*

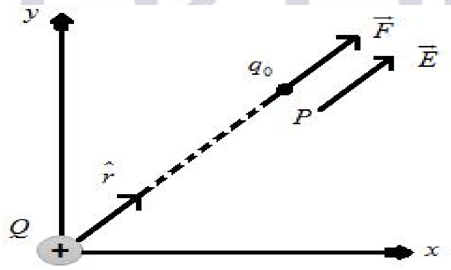
Calculation of electric field at a point: It is the measurement of force experienced by a one coulomb positive charge placed at that point.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Here q_0 is the test charge.

25)Can a gravitational field be added vectorially to an electric field?

Electric field due to a point charge



$$F = \frac{K\,q\,q_0}{r^2}$$

$$\frac{F}{q_0} = \frac{K\,q\,q_0}{r^2}$$

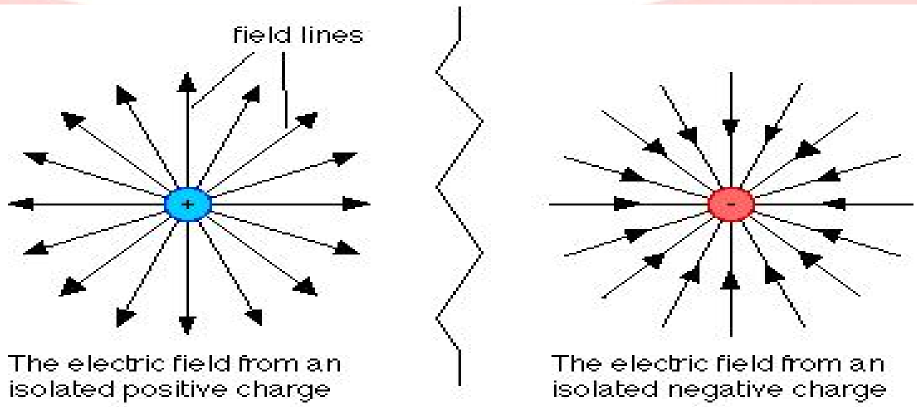
$$\frac{F}{q_0} = \frac{K\,q}{r^2}$$

$$E = \frac{K\,q(Source)}{r^2}$$

Note:-If we select positive test charge then field is always in the direction of force & vice-versa.

Electric field is a vector quantity.

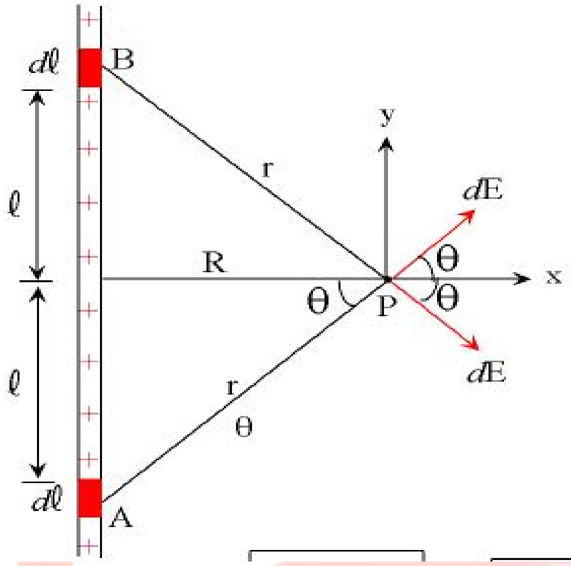
Direction of electric field:

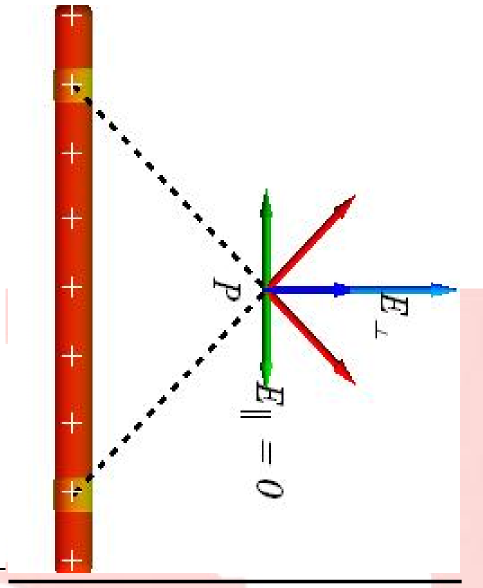


- Due to positive charge, electric field is radially out ward.

- Due to negative charge, electric field is radially in ward.

Electric field due to an infinite charged rod:





$$E_{net} = \int d\,E\cos\theta$$

$$E_{net} = \int \frac{k\,\lambda\,dl}{(l^2+R^2)}.\cos\theta$$

Here $\tan\theta = \frac{l}{R},\ l = R\tan\theta$.........(i)

$On\ Differentiating\ both\ sides\ w.r.t.\ \theta$

$$\frac{dl}{d\theta} = R\ sec^2\theta$$

$$dl = R\ sec^2\theta\ (dQ)$$ ....(ii)

$$E_{net} = \int \frac{k\,\lambda\,R\,sec^2\theta\ (d\theta)}{(R^2+l^2)}.\cos\theta$$

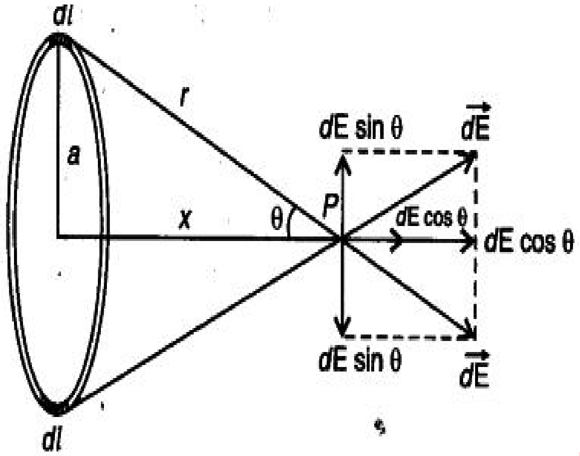
$$E_{net} = \frac{k\,\lambda}{R}\int_{-\pi/2}^{\pi/2}\cos\theta\ (d\theta)$$

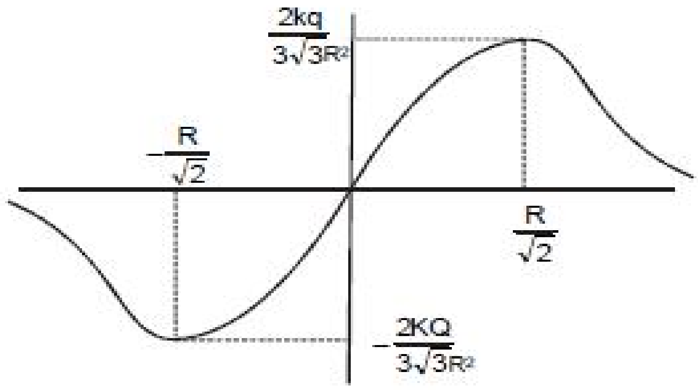
$$E_{net} = \frac{k\,\lambda}{R}\,[\sin\theta]_{-\pi/2}^{\pi/2}$$

$$E_{net} = \frac{2k\,\lambda}{R}$$

$$E_{net} = \frac{1}{2\pi\epsilon_0}.\frac{\lambda}{R}$$

Electric field due to ring at a point in its axis:





$$E_{net} = \int d\,E\,\cos\theta$$

$$E_{net} = \int_0^q \frac{1}{4\pi\epsilon_0}\cdot\frac{dq}{(R^2+x^2)^{3/2}}\cdot\frac{x}{\sqrt{R^2+x^2}}$$

$$E_{net} = \frac{1}{4\pi\epsilon_0}\cdot\frac{qx}{(R^2+x^2)^{3/2}}$$

$$here\ R = radius\ of\ ring$$

x = distance of point an the axis from centre

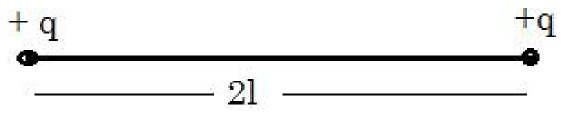
Electric Dipole: A system of any two equal & opposite point charges separated by a very small distance is known as electric dipole.

Ideal dipole:

1) Charge of the dipole must be as large as possible.
2) Length of the dipole must be as small as possible.

26)It is said that the separation between the two charges forming an electric dipole should be small. Small compared to what?

Electric dipole moment $(\vec{P})$***:*** It is a physical quantity which represents all the physical properties of an electric dipole. It is a vector quantity. Its direction is always from along the axis of dipole. Magnitude of electric dipole moment is given by the product of charge of dipole and length of dipole.

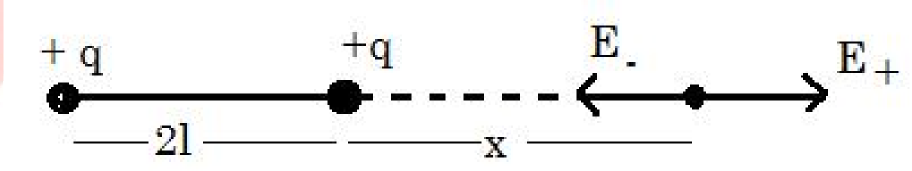


$|\vec{P}| = (q)\,(2l)$ or

$|\vec{P}| = 2ql$

Electric field due to dipole:

Case1:- Electric field on the axis of the dipole:



$$E_+ = k\frac{q}{(x-l)^2}$$

$$E_- = k\frac{q}{(x+l)^2}$$

$$\vec{E}axis = \vec{E}_+ + \vec{E}_-$$

$$Eaxis = E_+ - E_-$$

$$Eaxis = kq\left[\frac{1}{(x-l)^2} - \frac{1}{(x+l)^2}\right]$$

$$Eaxis = kq\,\frac{x^2 + l^2 + 2xl - x^2 - l^2 + 2xl}{(x^2 - l^2)^2}$$

$$Eaxis = kq\,\frac{4xl}{(x^2 - l^2)^2}$$

$$Eaxis = \frac{kq\,4xl}{(x^2 - l^2)^2}$$

$$Eaxis = \frac{k2px}{(x^2 - l^2)^2}$$

Magnitude of electric field due to short dipole on the axis $Eaxis = k\frac{2p}{x^3}$

Vector form of electric field due to short dipole on the axis $\vec{E}axis = k\frac{2\vec{p}}{x^3}$

Case-2:- Electric field on the equatorial line on the dipole

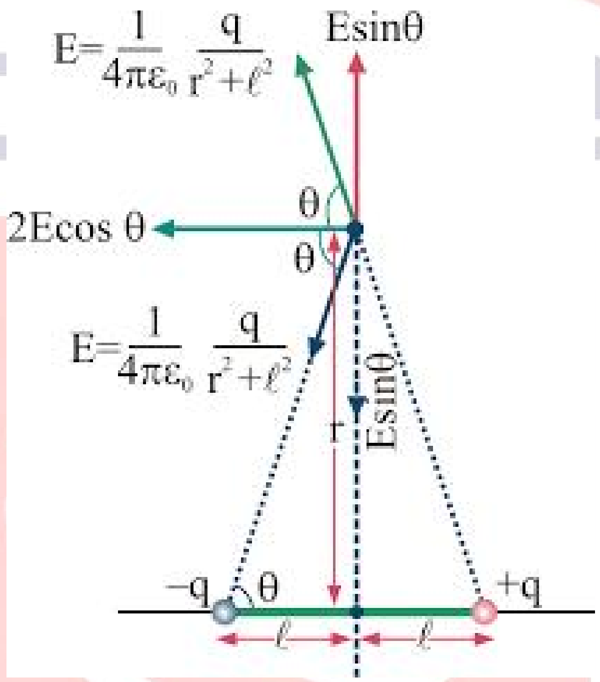
$E_{+} = E_{-} = \frac{k\,q}{(x^2 - l^2)}$

$Enet = 2k\,\frac{q}{(x^2-l^2)} \cdot \frac{l}{\sqrt{x^2-l^2}}$

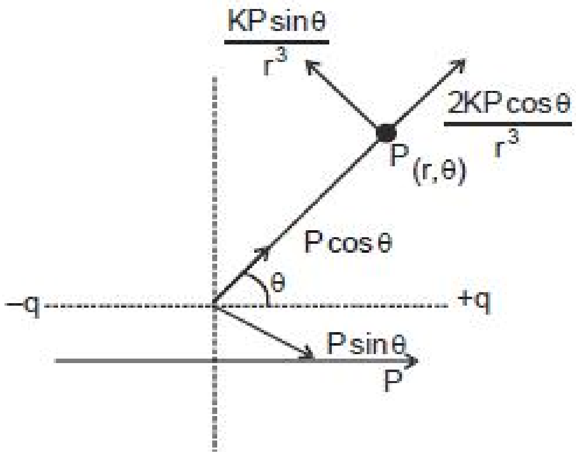
$$Enet = k\,\frac{p}{(x^2 - l^2)^{3/2}}$$

Magnitude of electric field due to short dipole on the equator $Enet = k\,\frac{p}{x^3}$

Vector form of electric field due to short dipole on the equator $\vec{E}net = -k\,\frac{\vec{p}}{x^3}$



Note:- General equation of electric field due to an electric dipole:



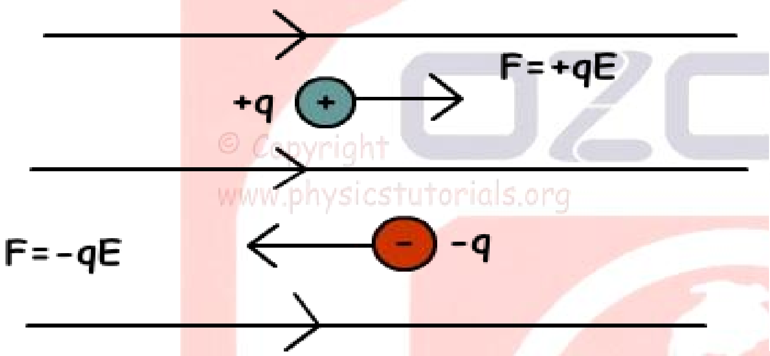
$$E_{net} = \sqrt{E_{\perp}^2 + E_{\parallel}^2}$$

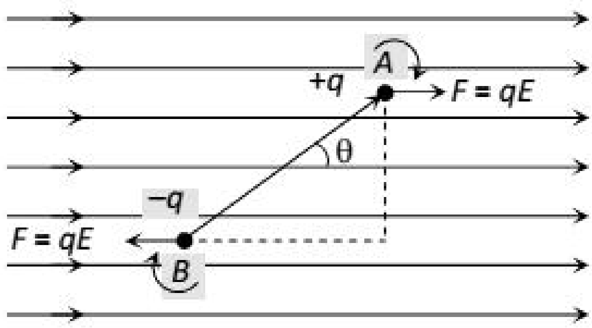
$$E_{net} = \sqrt{\left(\frac{kP\sin\theta}{r^3}\right)^2 + \left(\frac{2kP\cos\theta}{r^3}\right)^2}$$

$$E_{net} = \frac{kP}{r^3}\sqrt{\sin^2\theta + 4\cos^2\theta}$$

$$E_{net} = \frac{kP}{r^3}\sqrt{1 + 3\cos^2\theta}$$

Dipole in a uniform electric field:





Force on positive charge is always along the electric field

Force on negative charge is always opposite to electric field

$\sum \vec{F} = 0 \Rightarrow COM\ of\ system\ is\ in\ rest.$

$\sum \vec{\tau} \neq 0 \Rightarrow$ there is rotation

It means, in uniform electric field dipole shows rotation only.

Torque experienced by dipole in uniform electric field: Let at any instant dipole is at an angle θ with the field as shown then-

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$$

$\tau = \tau_1 + \tau_2$ (because both the torques are in same directions)

$$\tau = (qE)l\sin\theta + (qE)\,l\sin\theta$$

$$\tau = 2qEl\sin\theta$$

Magnitude of torque: $\boldsymbol{\tau = PE\sin\theta}$

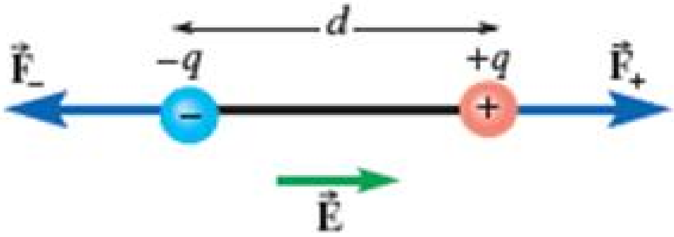
Vector form of torque $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{P}} \boldsymbol{X}\ \vec{\boldsymbol{E}}$

$$\tau_{max} = PE \qquad at\ \theta = 90^\circ$$

$$\tau_{min} = 0 \qquad at\ \theta = 0^\circ$$

Stable equilibrium of dipole in uniform electric field (When θ=0):

When we rotate the dipole slightly from its stable equilibrium position and release then it returns in its original position.



In stable equilibrium there is always a possibility of SHM.

When we rotate the dipole slightly from its stable equilibrium and release then it performs SHM & its time period can be calculated as following:

$$\sum \vec{F} \neq 0$$

$$\sum \vec{\tau} \neq 0$$

$$\tau = PE \sin\theta$$

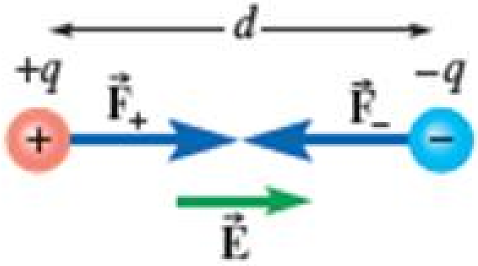
$$\tau = PE\theta$$

$$T = 2\pi\sqrt{\frac{I}{PE}}$$

Here I is moment of inertia of dipole.

Unstable equilibrium of dipole in uniform electric field (When θ=180^0):

When we rotate the dipole slightly from its stable equilibrium position and release then it does not return to its original position.



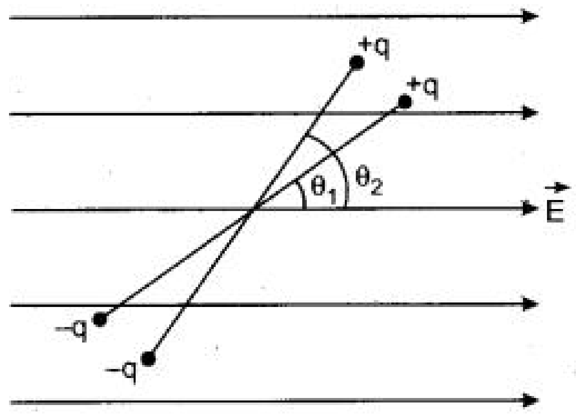
Work done to rotate the dipole in external electric field: Let at any instant dipole is at an angle θ with the field & we further rotate it by a very small angle $d\theta$ as shown, then –

$$W = \int \tau\ (d\theta)$$

$$W = \int_{\theta_1}^{\theta_2} PE \sin\theta\ (d\theta)$$

$$W = -PE\ [\cos\theta]_{\theta_1}^{\theta_2}$$

$$W_{ext} \Rightarrow PE\ [\cos\theta_1 - \cos\theta_2]$$



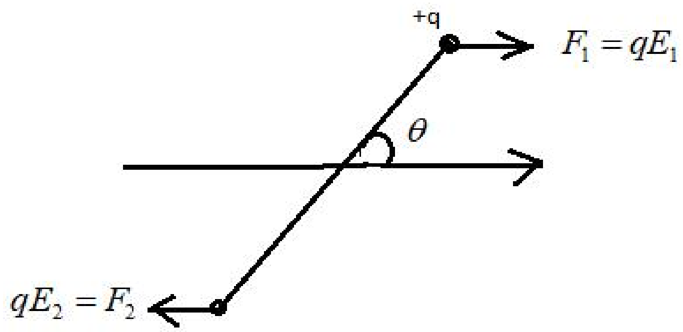
The diagram of work done in rotating a electric dipole in uniform magnetic field

Dipole in non-uniform electric field:

In this case dipole experiences a net force and net torque both, hence it shows translational motion and rotational motion both.long

$$\Sigma \vec{F} \neq 0$$

$$\Sigma \vec{\tau} \neq 0$$



Net force on dipole is $Fnet = qE_1 - qE_2$ [When $E_1 > E_2$]

Net torque on dipole is $\tau_{net} = qE_1 l\sin\theta + qE_2 l\sin\theta$

$$\tau_{net} = q\, l\sin\theta\, (E_1 - E_2)$$

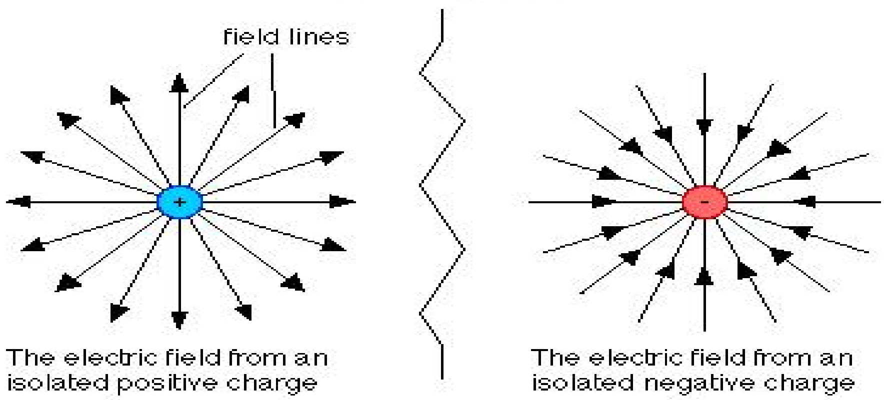
$$\tau_{net} = \frac{P}{2}\sin\theta\, (E_1 - E_2)$$

Electric field lines (EFL): These are imaginary lines drawn in an electric field, by which we can represent relative magnitude & direction of electric field at any point.

Note: By EFL we can determine direction of force on a charge also, hence electric field lines are called electric line of force also. (In case of magnetic field lines this is not possible)

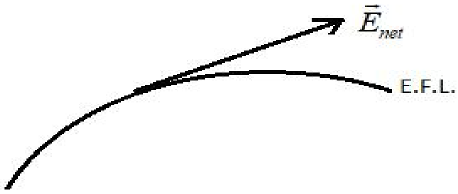
Properties of electric Field lines:

1. EFL are produced from positive charge or from ∞.
2. EFLs are terminate at negative charge or at ∞.

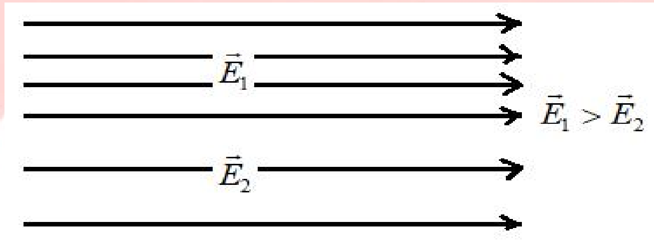


Note: EF lines always represent net electric field & at a single point net electric field has only one direction.

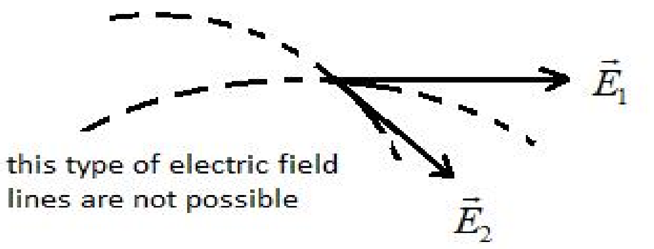
3. Tangent drawn at a point on the electric field line represents direction of net electric field at that point.



4. Closeness of EFLs is directly proportional to magnitude of net electric field.

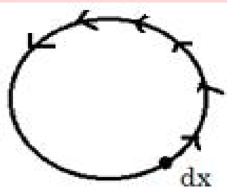


5. Two different lines never intersect because at a single point, there cannot be two different directions of net electric field.



6. EFLs never form closed loop because electric field is a conservative field.

Note: In a conservative field, net work done in a closed loop must be zero while if EFLs form closed loop then net work done will not be zero, hence closed loop of lines is not possible. (different from MFLs)

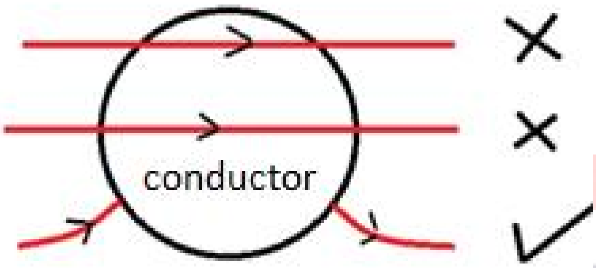


$dw = qE\,(dx\cos 0)$ [in this situation θ is always 0^0 hence net work done can not be zero]

$dw = qE\,dx \neq 0$

7. Inside a conductor, EFLs are not possible because net electric field inside a conductor is always 0.

8. At the surface of a charged conductor EFLS must be always normal.

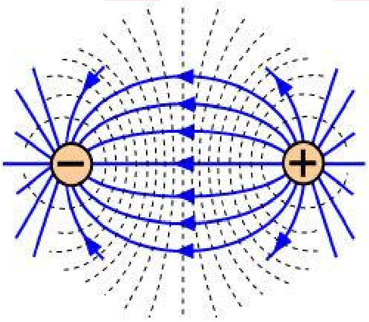


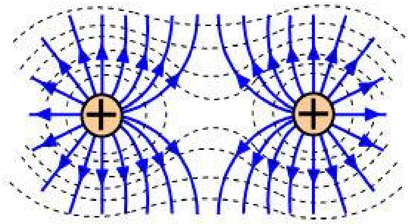
9. Number of lines are directly properties to amount of charge.

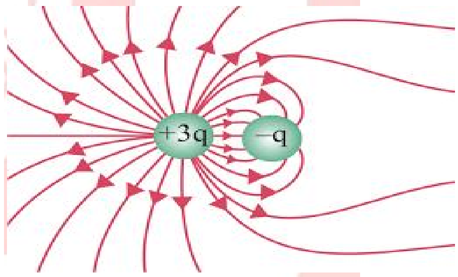
$$N \propto q$$

$$\frac{N_1}{N_2} = \frac{q_1}{q_2}$$

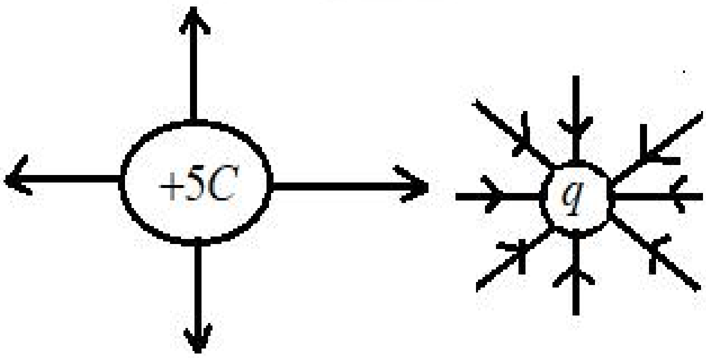
Some important electric field lines:







27) In the given diagram find q?



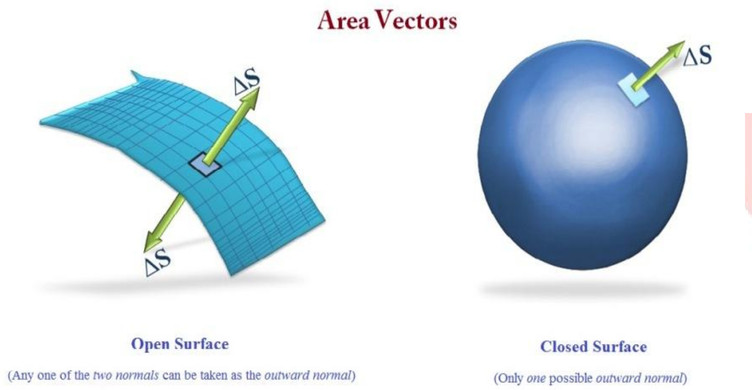
28) A charge +q is placed in the cavity of a conductor, draw the electric field lines.

29) Draw electric field line for an electric dipole.

ELECTROSTATICS-2

Area Vector: $\vec{A} = |\vec{A}|\ \hat{A}$ It must be always ⊥ to the given area.

- For open area (square, rectangle, disc, ring) area vector may have both normal directions.
- For closed area (cube, sphere) area vector must be always outward normal direction only.



Electric Flux:- It is the measurement of number of EFLs penetrating through a given area. It is scalar quantity. Mathematically it is given by:

$$\emptyset = \int \vec{E} \, . \, d\vec{A}$$

$$\emptyset = \int E \, . \, (dA) \cos\theta,$$

If E is uniform, then

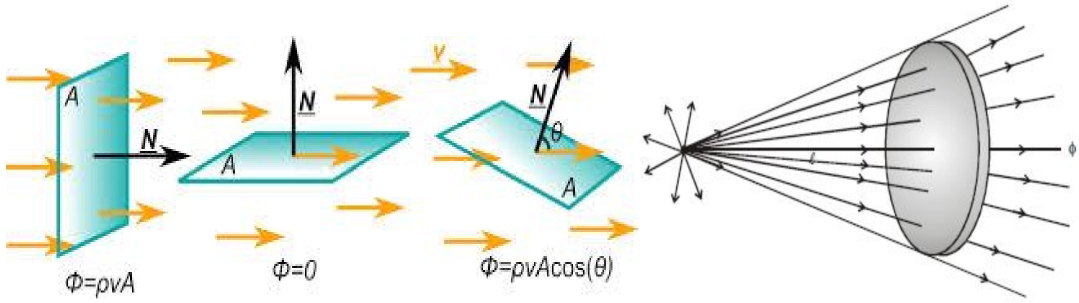
$$\emptyset = E \int (dA) \cos\theta$$

$$\emptyset = E A \cos\theta$$

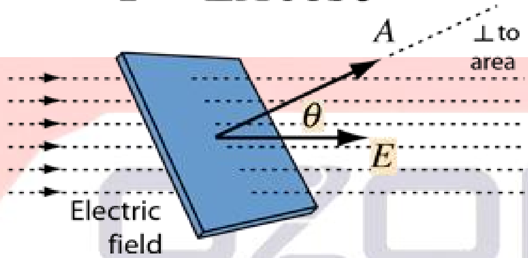
Here : A is area inside the field

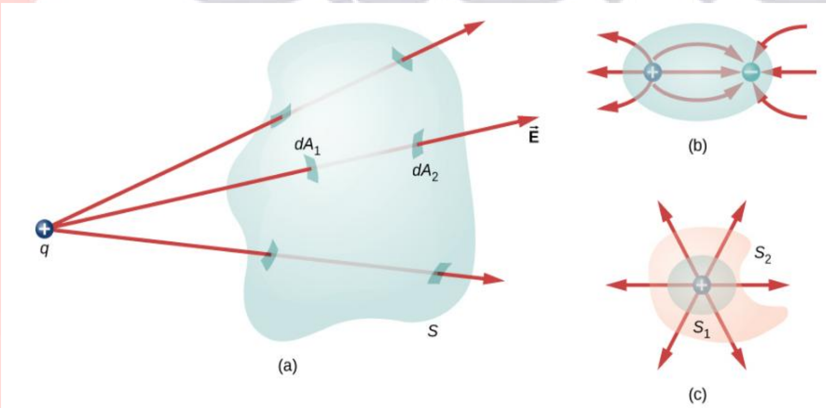
E is electric field at the given area.

θ is angel between area vector & electric field.

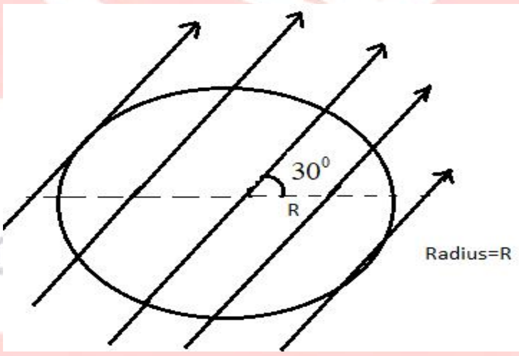


flux = $\Phi = EA\cos\theta$





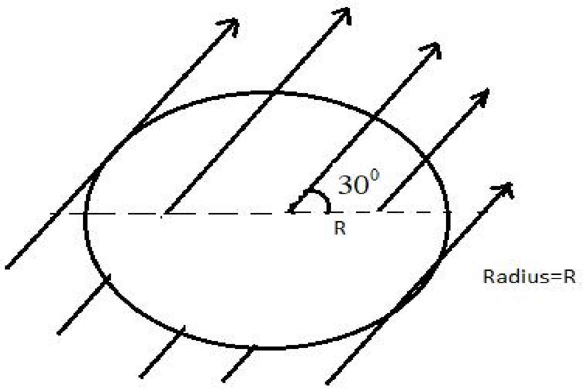
30)Find net flux through the given disc.



Sol: $\emptyset = EA\cos\theta$

$\emptyset = E_0(\pi R^2)\cos 90^\circ \Rightarrow \emptyset = 0$

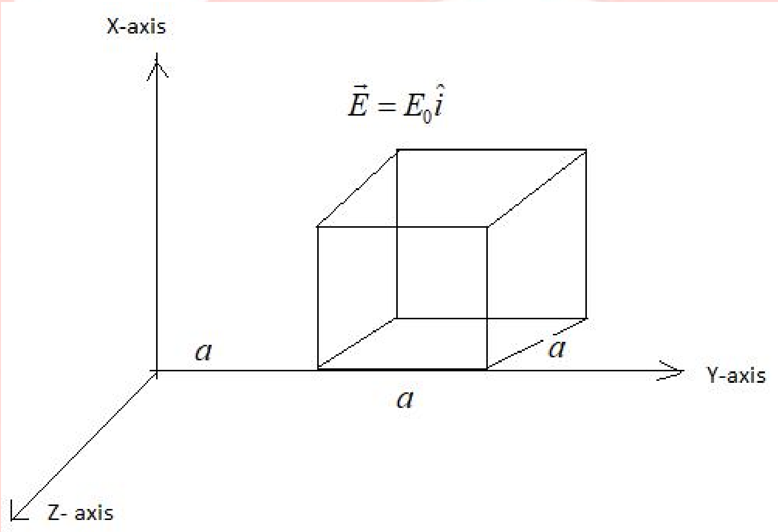
31)Find net flux in the given disc.



Sol: $\phi = EA\cos\theta$

$\phi = E_0(\pi R^2)\cos 60° \Rightarrow \phi = \frac{\pi R^2 E_0}{2}$

32)Find net flux through the given cube.



Sol: $\phi = \phi_1 + \phi_2$

$\phi = E_0a^2\cos 180° + E_0a^2\cos 0°$

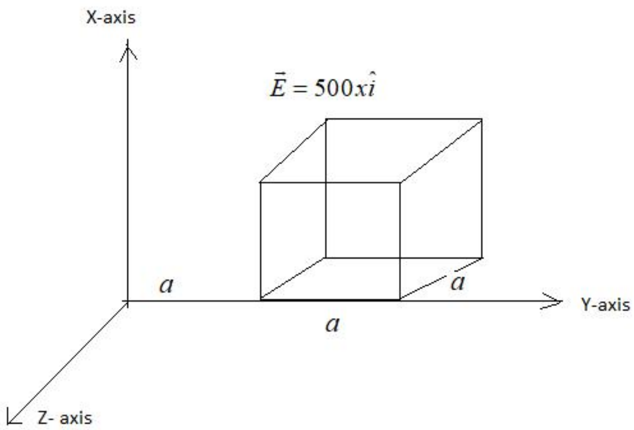
$\phi = -E_0a^2 + E_0a^2$

$\phi_{net} = 0$

Conclusion:

- Inward flux is always negative as $\theta = 180°$.
- Outward flux is always positive as $\theta = 0°$.
- If a closed body is placed in a uniform electric field then net flux is always 0.

33)Find net flux through the cube.



Sol.

$$\phi_{net} = \phi_1 + \phi_2$$

$$\phi_{net} = (500a)(a^2)\cos 180° + (500(2a)(a^2)\cos 0°$$

$$\phi_{net} = -500a^3 + 1000a^3$$

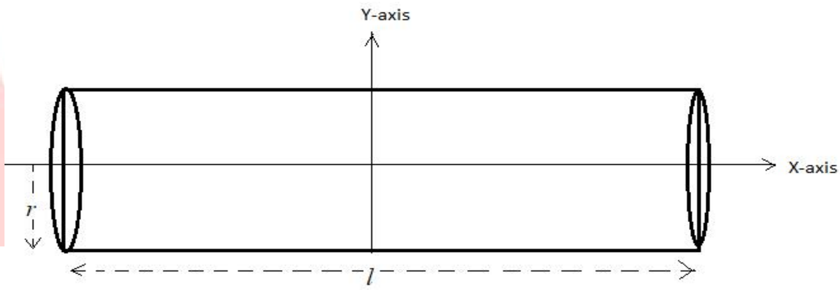
$$\phi_{net} = 500a^3$$

34)If electric field in a region is given by

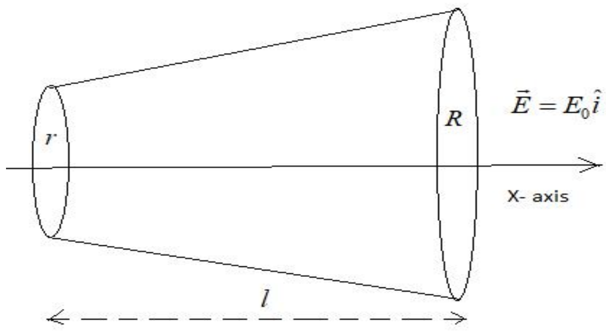
$$\vec{E} = 500\hat{i}, \quad x > 0$$

$$\vec{E} = -500\hat{i}, \quad x < 0$$

Then find out net electric flux through the given cylinder



35)Find flux linked to the curved surface of the cylinder.



Sol.

$$\phi_1 + \phi_2 + \phi_3 = 0$$

$$-(\phi_1 + \phi_2) = 0.\phi_3$$

$$-[(E_0)(\pi R^2)\cos 0° + E_0(\pi r^2)\cos 180°]$$

$$(-E_0\pi R^2 - E_0\pi r^2)$$

$$(-E_0\pi R^2 - E_0\pi r^2)$$

$$\phi_3 = \pi E_0[R^2 - r^2]$$

Gases Theorem:

According to Gauss' theorem net electric flux through a closed surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

$$\phi_{net} = \frac{1}{\varepsilon_0} q_{enclosed}$$

Or

Surface integral of net electric field over a closed surface is always equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

$$\oint EdA\cos\theta = \frac{1}{\varepsilon_0} q_{enclosed}$$

- *Gases theorem is applicable for any type of closed surface either symmetrical or unsymmetrical w.r.t. the given charge distribution but*
- *Practically we use Gauss' theorem for the symmetrical surfaces only where electric field is uniform over the surface or zero [to make calculations simple]*

Gaussian Surface:

Any closed surface for which we apply Gauss' theorem is called Gaussian surface.

In practice we select highly symmetrical Gaussian surface only (Cube, Sphere, Cylinder)

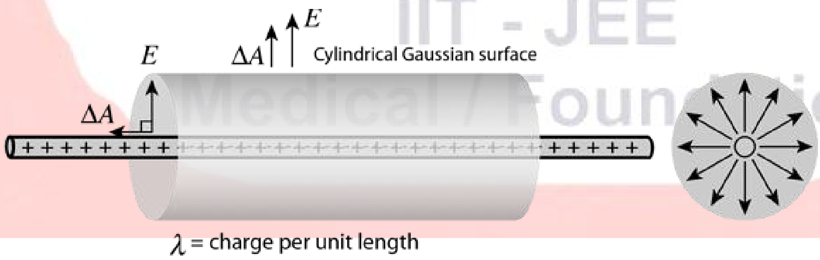
36)A charge q is placed at a height of a/2 from centre of square plate of side a. Find net flux through the plate.

Sol: If we assume a cube of side a, then the charge is at its centre and $\emptyset_{net}$ through the cube is q/ε_0 which is equally divided between all the 6 surfaces. Hence, flux through the one face is

$$\phi = \frac{q}{6\varepsilon_0}$$

37) If a charge q is placed at the corner of a cube of side a then find net flux through the cube.

Calculation of Electric field due to an infinite charge rod:



Let a cylindrical Gaussian surface of radius r as shown:

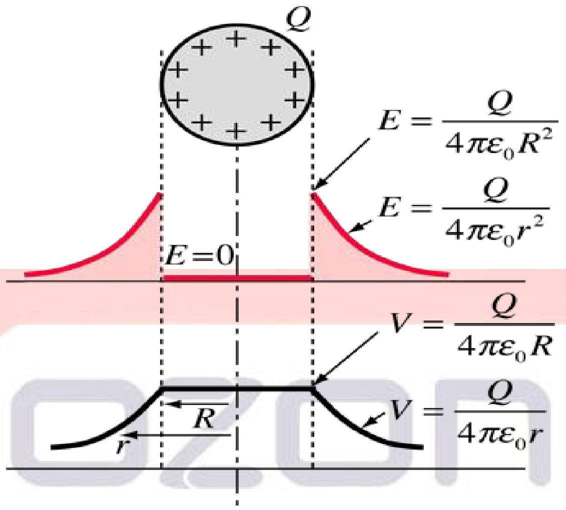
$$\oint E(dA)\cos\theta = \frac{q\,in}{\epsilon_0}$$

$$\oint E(dA)\cos 0^\circ + \oint E(dA)\cos 90^\circ + \oint E(dA)\cos 0^\circ$$

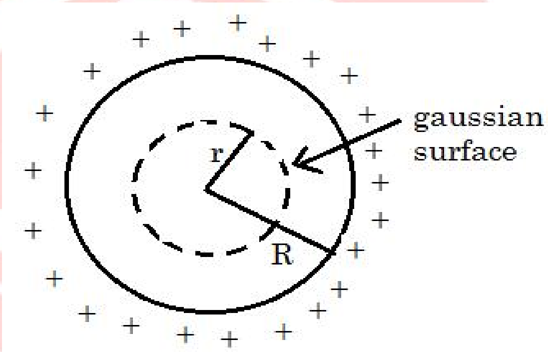
$$E = (2\pi\, r l) = \frac{\lambda\, l}{\epsilon_0}$$

$$E = \frac{1}{2\pi\epsilon_0}.\frac{\lambda}{r}$$

Electric field due to charged spherical shell or conducting solid sphere:



Case-1: At a point inside (r<R)

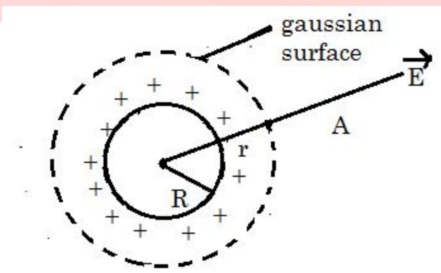


$$\oint E(dA)\cos\theta = \frac{q\,in}{\epsilon_0}$$

$$E\oint dA\cos\theta = \frac{0}{\epsilon_0}$$

E = 0

Case-2: At a point out side (r>R)



$$\oint E(dA)\cos\theta = \frac{q\,in}{\epsilon_0}$$

$$E\oint dA\cos 0^\circ = \frac{q}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{q}{\epsilon_0}, \quad E \propto \frac{1}{r^2}$$

$$E_{out} = \frac{1}{4\pi\epsilon_0}.\frac{q}{r^2}$$

Case-3: At a point on the surface (r = R)

$$E_{surface} = \frac{1}{4\pi\epsilon_0}.\frac{\theta}{R^2}$$

Electric field due to charged infinite sheet:

Infinite sheet:- Distance of point from sheet is very small as compared to size of sheet.

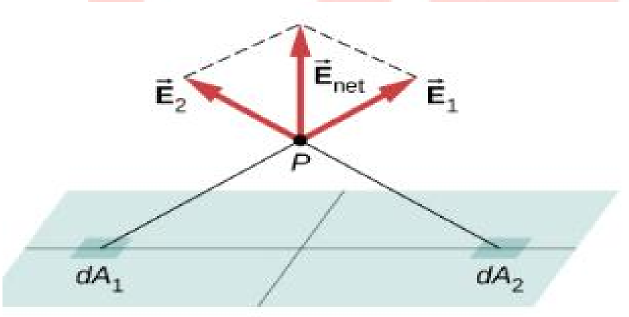
$$\oint E(dA)\cos\theta = \frac{q\,in}{\epsilon_0}$$

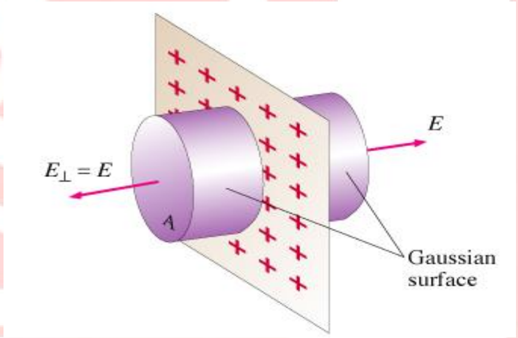
$$\int E(dA)\cos 0° + \int E(dA)\cos 90° + \int E(dA)\cos 0° = \frac{\sigma A}{\epsilon_0}$$

$$EA + 0 + EA = \frac{\sigma A}{\epsilon_0}$$

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$





Sheet: Charge of the one face is counted on other face also.

Plate: Charge on both the faces are different.

Electric field due to a charged infinite plate:

$$\oint E(dA)\cos\theta = \frac{q\,in}{\epsilon_0}$$

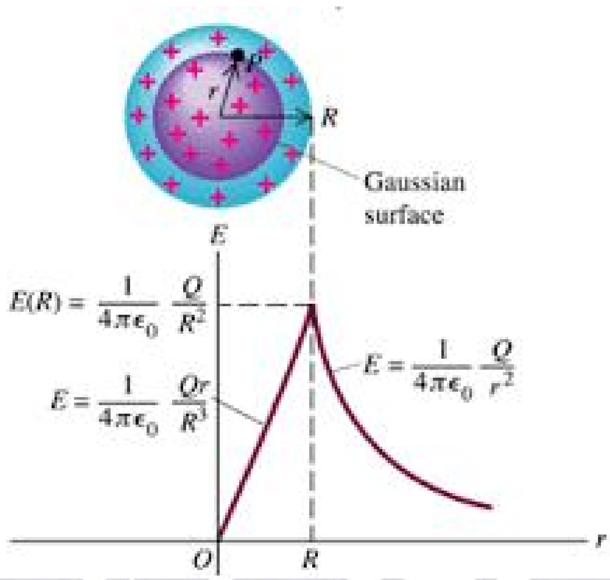
$$\int E(dA)\cos 0° + \int E(dA)\cos 90° + \int E(dA)\cos 0° = \frac{\sigma(2A)}{\epsilon_0}$$

$$EA + 0 + EA = \frac{2\sigma A}{\epsilon_0}$$

$$2EA = 2\frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$

Electric field due to a charged dielectric sphere:



Case-1: At a point outside:

$$\oint E(dA)\cos\theta = \frac{q\,in}{\epsilon_0}$$

$$\oint E(dA)\cos 0^\circ = \frac{q}{\epsilon_0}$$

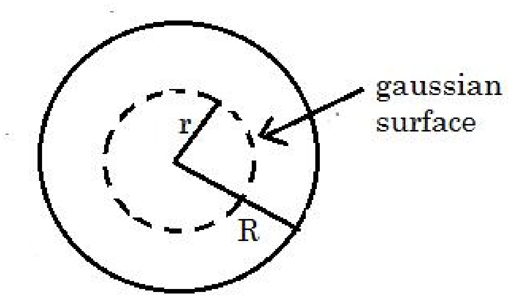
$$E(4\pi r^2) = \frac{q}{\epsilon_0}, \quad E \propto \frac{1}{r^2}$$

$$E_{out} = \frac{1}{4\pi\epsilon_0}.\frac{q}{r^2}$$

Case.2 At a point on the surface:

$$E_{surface} = \frac{1}{4\pi\epsilon_0}.\frac{q}{R^2}$$

Case.3: At a point inside



$$\oint E(dA)\cos\theta = \frac{q\,in}{\epsilon_0}$$

$$E\oint(dA)\cos 0^\circ = \rho\left(\frac{4}{3}\pi r^3\right)$$

$$E(4\pi r^2) = \frac{\rho}{\epsilon_0}\left(\frac{4}{3}\pi r^3\right)$$

$$E_{inside} = \frac{1}{3}\frac{\rho}{\epsilon_0}r$$

$$E_{inside} = \frac{q}{4\pi\epsilon_0 R^3}.r$$

Properties of conductor in electrostatic condition (when current through the conductor is zero):

1. Net electric field inside the conductor must be zero, otherwise there will be a current inside the conductor due to the flow of free electrons.
2. Excess charge of conductor resides on its surface but not inside it.

$$\oint E(dA)\cos\theta = \frac{q\ in}{\epsilon_0}$$

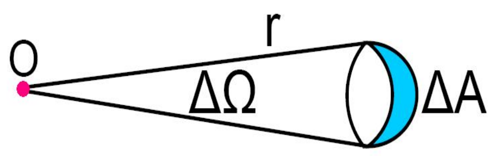
$$0 = \frac{q\ in}{\epsilon_0}$$

$$q\ in = 0$$

3. Electric field at every point on the surface of a charged conductor must be normal to the surface otherwise there will be a current on the surface of conductor due to the tangential component of electric field which is not possible.
4. Electric potential at every point inside a charged conductor must be constant and that is equal to the potential on the surface of the conductor.

<u>Solid angle:</u>

Angle formed by an areal arc at a point is known as solid angle. It is an angle in 3D which represents relative size of given areas.



Mathematically, solid angle $\Omega = \frac{\perp area}{(radius)^2}$

$$\Omega = \frac{A\perp}{r^2}$$

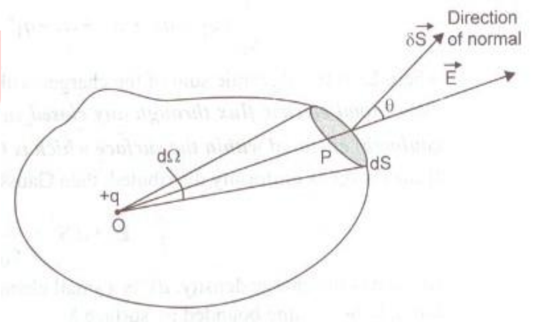
- For a sphere net solid angle is 4π steradian at its centre.

- Net solid angle formed by any type of closed body at a point inside the body is always equal to 4π steradian.

$$\oint \frac{dA\cos\theta}{r^2} = 4\pi$$

Proof of gauess theorem by Coulomb's law:

Let a charge "q" is enclosed by a closed body as shown in figure.



Then the electric flux through a very small element of the surface area of the closed body is given by $d\phi = E(dA)\cos\theta$

Net flux through the entire closed body is

$$\oint d\phi = \oint E(dA)\cos\theta$$

$$\phi = \oint \frac{1}{4\pi\epsilon_0}.\frac{q}{r^2}\ (dA)\cos\theta$$

$$\phi = \frac{1}{4\pi\epsilon_0}.q\oint\frac{dA\cos\theta}{r^2}$$

$$\phi = \frac{1}{4\pi\epsilon_0}.q\ (4\pi)$$

$$\phi = \frac{q}{\epsilon_0}$$

In general form, $\oint E\ (dA)\cos\theta = \frac{q_{in}}{\epsilon_0}$

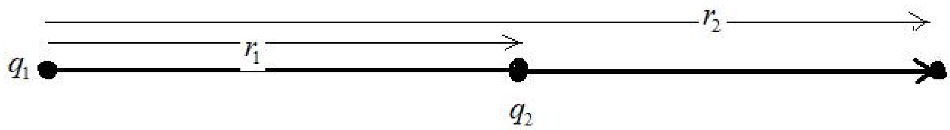
Earthing: Potential of an earthed body must be equal to the potential of the earth i.e. conventionally equal to zero.

⇒ Charge of an earthed body may or may not be zero but its potential is always zero

Electric Potential and Electric Potential Energy

Let's start with a question which is in fact backbone of this whole concept.

38)In the given diagram, find the work done by electric force to displace the charge q_2 from position r_1 to r_2.

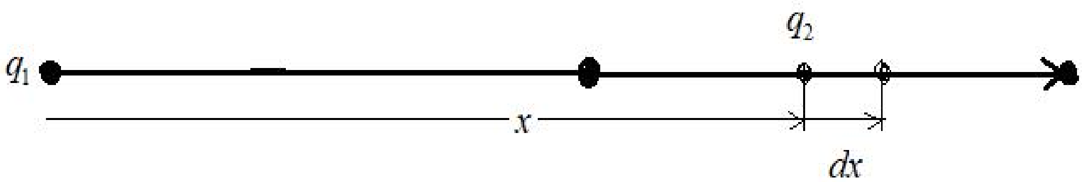


OR

Prove that electric force is a conservative force.

Sol: Work done by F_e to bring a charge from r_1 to r_2.

$$W_{int} = \int F(dx)\cos\theta$$



$$W_{int} = \int_{r_1}^{r_2} k\frac{q_1 q_2}{x^2}(dx)\cos 0^\circ$$

$$W_{int} = k\, q_1 q_2 \int_{r_1}^{r_2} \frac{1}{x^2} dx$$

$$W_{int} = -k\, q_1 q_2 [\frac{1}{x}]_{r_1}^{r_2}$$

$$W_{int} = -k\, q_1 q_2 [\frac{1}{r_2} - \frac{1}{r_1}]$$

$$\boldsymbol{W_{int} = k\, q_1\, q_2 [\frac{1}{r_1} - \frac{1}{r_2}]}$$

Note:-

(i) Above equation of work done is independent from the path followed by the charge. It only depends on initial and final position. It proves that electric force is a conservative force.

(ii) From above equation we can't directly identify that which of the charges or both the charges were displaced, it means if we change the position of a system of two charges anyhow, we can use above formula in all the cases to determine work done by electric force.

Concept of Potential energy:

(i) Concept of potential energy is not compulsory in physics, but it is only necessary to make calculation of work done by variable conservative forces in simple way.

(ii) Potential energy is only defined in conservative field.

(iii) We cannot determine absolute potential energy at any point but always difference of potential energy between two points remains constant, which can be calculated.

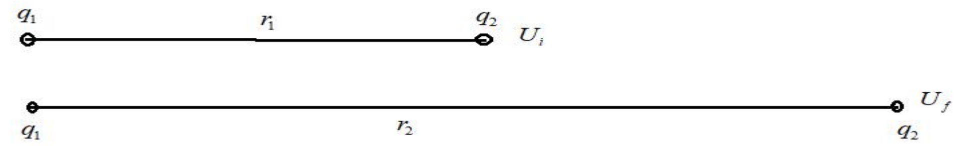
(iv) As per our convenience we can select zero potential energy at any selected reference point.

Note:- In conservative field, always $\boldsymbol{W_{ext} \Rightarrow - W_{int}}$ [When we displace a system without acceleration].

Calculation of potential energy: Always negative of work done by net conservative force is equal to change in potential energy of a system, to displace from one point to another point.

$$\boldsymbol{\Delta U = -W_{conservative}}$$

Electric Potential energy of two point charge system:



$$\Delta U = W_{ext}$$

$$\Delta U = k\, q_1 q_2 \left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

if $r_1 = \infty \;\&\; r_2 = r$ then

$$U_r - U_\infty = k\, q_1 q_2 [\frac{1}{r} - \frac{1}{\infty}]$$

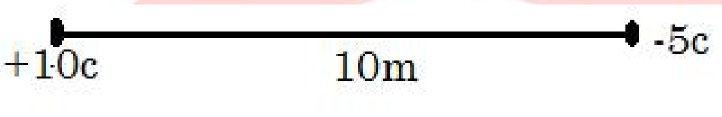
$$\boldsymbol{U_r - U_\infty = k\, q_1\, q_2 \frac{q_1 q_2}{r}}$$

If, $U_\infty = 0$, then $\boldsymbol{U_r = \frac{k\, q_1 q_2}{r}}$

Note:

- *Use above equation with sign.*
- *Above proof of potential energy is not acceptable in board exams. This method is only for real understanding of electric potential energy.*

39)Find out P.E. of the given system.



Sol. $$U = \frac{k\, q_1 q_2}{r}$$

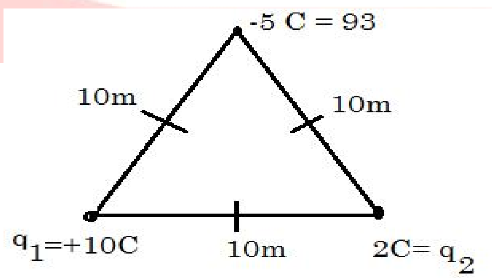
$$U = \frac{(10)(-5)}{10} = -5k\ \ J$$

40)When the separation between two charges is increased, the electric potential energy of the charges:

(a)increase (b)decreases (c)remains the same (d)may increase or decrease

Ans: d

41)Find P.E. of the given system.



Sol. $$U = \frac{k\, q_1 q_2}{r_{12}} + \frac{k\, q_1 q_3}{r_{13}} + \frac{k\, q_2 q_3}{r_{23}}$$

$$U = k\ [\frac{20}{10} + \frac{-50}{10} + \frac{-10}{20}] = 2 - 5 - 1 = 2 - 6 = -4k$$

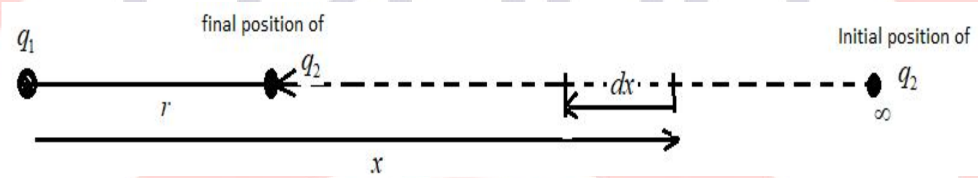
Proof of electric potential energy of two points charge system: [For board exams]

Net work done by an external agent to form a system, on bringing each charge from infinity one by one without acceleration is the measurement of potential energy of the system.

Let a charge q_1 is fixed and we bring an other charge q_2 from ∞ to position r w.r.t. q_1 then net work done by external agent in this process is measured by following way:

Work done by F_{ext} to bring the charge q_2 from ∞ to r without acceleration is:

$$W = \int F(dx)\cos\theta$$



$$W_{ext} = \int_{\infty}^{r} k\frac{q_1 q_2}{x^2}(-dx)\cos 0^\circ$$

$$W_{ext} = -k\,q_1 q_2 \int_{\infty}^{r}\frac{1}{x^2}dx$$

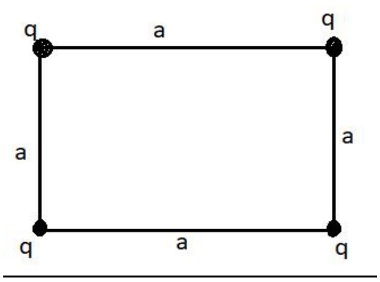
$$W_{ext} = k\,q_1 q_2\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$W_{ext} = k\,q_1 q_2\left[\frac{1}{r}-\frac{1}{\infty}\right]$$

$$\boldsymbol{W_{ext} = k\frac{q_1 q_2}{r}}$$

$$\boldsymbol{U = k\frac{q_1 q_2}{r}}$$

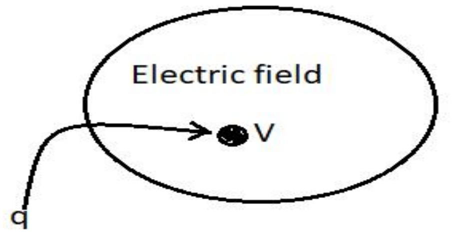
42)Find potential energy of the given charge system.



Electric Potential at a point:

1. Electric potential is a physical quantity which decides energy of a charge particle placed at a point in a given electric field.
2. Electric potential is produced by a source charge at every point in its surrounding.
3. Electric potential due to a charge, at a point where charge is placed is always zero.
4. It is scalar quantity. Its SI unit is volt (V).

Let a charge q is placed at a point where electric potential is V then energy of the charge is given by



U=qV

After partial differentiation

dU=q(dV)

If we displace the charge q between two points without acceleration then

$$dW_{ext}=q(dV)$$

$$dV=\frac{dW_{ext}}{q}$$

It means work done by external agent to displace a unit positive charge without acceleration between two points is the measurement of potential difference between the both points.

43)When the separation between two charges is increased, the electric potential energy of the charges: [Revision question]

(a)increase (b)decreases (c)remains the same (d)may increase or decrease

Ans: d

44)If a +ve charge is shifted from a low potential region to a high potential region, the electric potential energy:

(a)increase (b)decreases (c)remains the same (d)may increase or decrease

Ans: a

Calculation of electric potential difference between two points: Work done by external agent to displace a unit positive charge from one point to other point is equal to potential difference between both the points.

$$\Delta V = \frac{W_{ext}}{q}$$

$$V_f - V_i = \frac{W_{ext}}{q}$$

$$V_f - V_i = \frac{\int dW_{ext}}{q}$$

***Electric potential due to a point charge*:**

$$\Delta V = \frac{W_{ext}}{q}$$

$$V_f - V_i = \frac{Qq}{q}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

$$V_r - V_\infty = kQ\left(\frac{1}{r} - \frac{1}{\infty}\right)$$

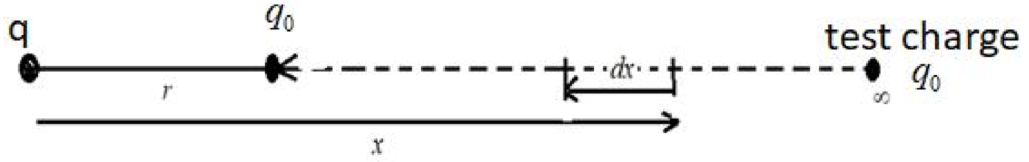
$$V = k\frac{Q_{source}}{r}$$

Note: Use above equation with sign.

We will use frequently $W_{ext} = q\Delta V$ in later discussion

Calculation of electric potential due to a point charge system:

Electric potential at a point is the measurement of work done by external agent to bring a unit positive charge from ∞ to the given point.



$$W_{ext} = \int F dx \cos\theta$$

$$W_{ext} = \int_{\infty}^{r} k\frac{q q_0}{x^2}(-dx)\cos 0°$$

$$W_{ext} = -kq\, q_0 \int_{\infty}^{r} \frac{1}{x^2} dx$$

$$W_{ext} = kq\, q_0 [\frac{1}{x}]_{\infty}^{r}$$

$$W_{ext} = kq\, q_0 [\frac{1}{r}]$$

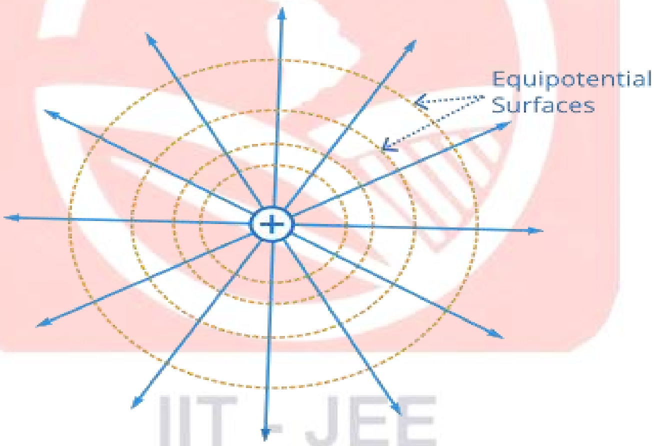
$$\frac{W_{ext}}{q_0} = \frac{kq}{r}$$

$$\boldsymbol{V = \frac{kq}{r}}$$

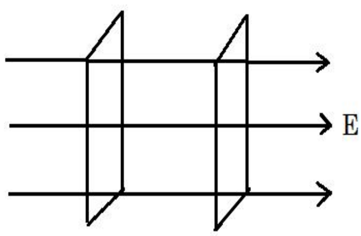
Equi -Potential surface:

A surface which has same potential at every point lying on it is called equipotential surface.

1. Due to point charge, equipotential surfaces are concentric spheres.



2. Due to line charge, equipotential surfaces are coaxial cylinders. (Co-axial cylinder).
3. In a region of uniform electric field equipotential surfaces are parallel planes(due to planer source).



Note: *A very small portion of a very large spherical or cylindrical Gaussian surface also acts as planer Gaussian surface.*

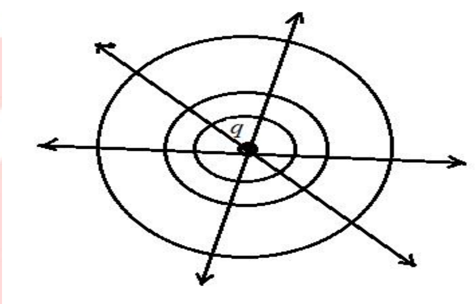
Properties of equipotential surface:

1. Net work done to displace a charge from one point to other point on any equipotential surface is always zero.

$$W = q(\Delta V)$$

$$W = q(V - V) = 0$$

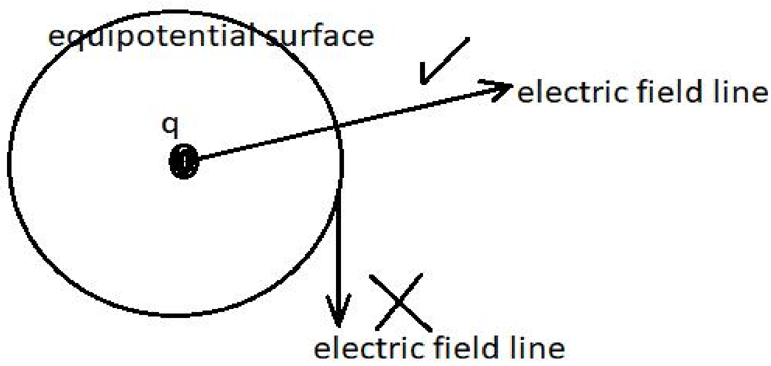
2. Closeness of equipotential surfaces is directly proportional to the strength of electric field.



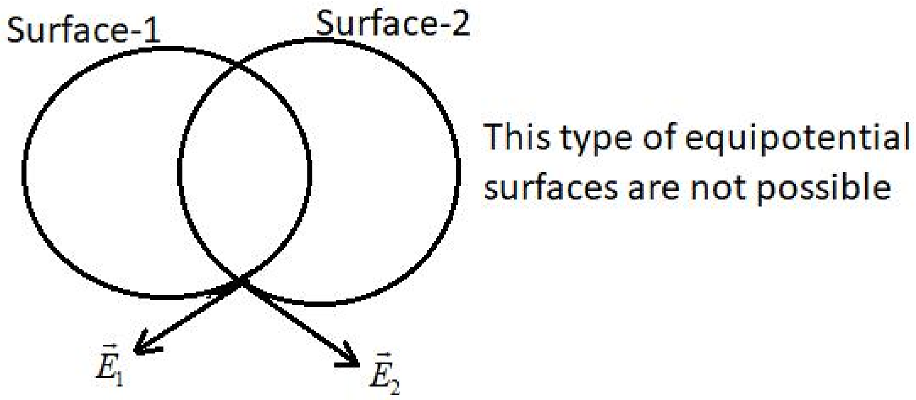
3. Electric field is always normal to every point of an equipotential surface.

Or

Component of electric field along the equipotential surface is always zero.

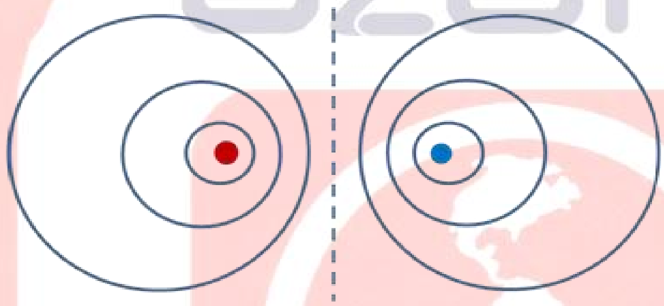


4. Different equipotential surfaces never intersect because at a single point there cannot be two different directions of net electric field.



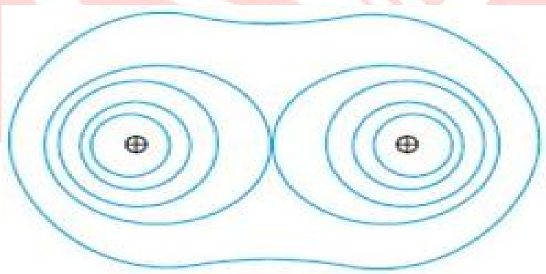
45)Plot the equipotential surfaces due to an electric dipoles.

Sol.



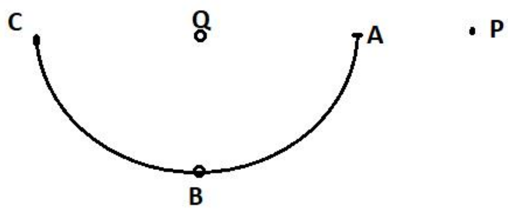
46)Plot the equipotential surfaces due to two equal and like charges.

Sol.



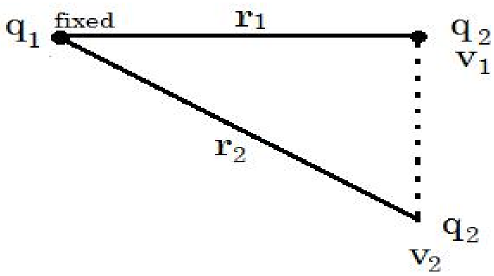
***47)*Consider the situation as shown in fig. The work done in taking a point charge from P to A is W_A, from P to B is W_B, and from P to C is W_c then:**

(a) $W_A < W_B < W_C$ **(b)** $W_A > W_B > W_C$ **(c)** $W_A = W_B = W_C$ **(d) None**



Ans: C

Relation between work done and potential difference:



$$U_i = \frac{k\,q_1 q_2}{r_1}$$

$$U_j = \frac{k\,q_1 q_2}{r_2}$$

$$W_{ext} = \Delta U$$

$$W_{ext} = U_f - U_i$$

$$W_{ext} = \frac{k\,q_1 q_2}{r_2} - \frac{k\,q_1 q_2}{r_1}$$

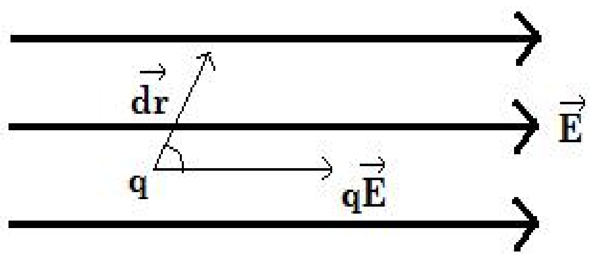
$$W_{ext} = q_2 (\frac{k\,q_1}{r_2} - \frac{k\,q_1}{r_1})$$

$$W_{ext} = q_2\,(V_f - V_i) = q_2\,(\Delta V)$$

It means work done to displace a charge q from one point to the other point is

In general, $\boldsymbol{W_{ext} = q\,(\Delta V)}$

Relation between electric field and electric potential:



$$dw_{int} = qE(dr)\cos\theta$$

$$-dw_{ext} = qE(dr)\cos\theta$$

$$-q(dV) = qE(dr)\cos\theta$$

$$-dV = E(dr)\cos\theta$$

$$\Delta V = -\int E(dr)\cos\theta$$

$$\Delta V = -\int \vec{E}.\vec{dr}$$

$$E = \frac{-dv}{dr\cos\theta}$$

$$\boldsymbol{E = \frac{-dv}{dr}}$$ **When θ=0 or displacement is along the field.**

Negative sign, represents that potential decreases in the direction of electric field.

<u>Relation between Kinetic energy and potential difference:</u>

Method-1: By work energy them:

$$\Delta K = W_{ext}$$

$$K_f - K_i = W_{ext}$$

If initial kinetic energy is zero then:

$$K_f - O = W_{ext}$$

$$K = W_{ext}$$

$$\boldsymbol{K_{max.} = q\Delta V} \quad \text{because}\{W_{ext} = q\,(\Delta V)\}$$

Method-2: By concept of force:

$$F = qE$$

$$ma = qE$$

$$a = \frac{qE}{m}$$

Equation of motion,

$$v^2_{max.} = o^2 + \frac{2qE}{m}(dr)$$

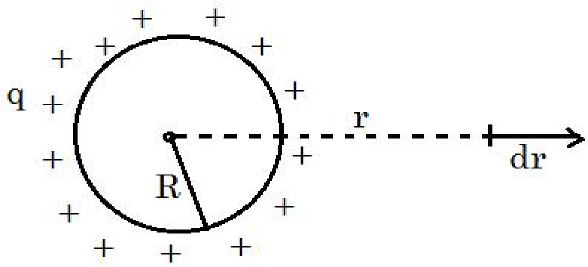
$$\frac{1}{2}mv^2_{max.} = qE\,(dr)$$

$$K_{max} = qE\,(dr)$$

$$K_{max} = q\,(\Delta V)$$

Calculation of electric potential due to a charged spherical shell:

a. ***At a point out side***



$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$E = \frac{-dV}{dr}$$

$$\int_0^V dV = -\int_\infty^r \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr$$

$$V = -\frac{1}{4\pi\epsilon_0}.q\int_\infty^x \frac{1}{r^2}\,dr$$

$$V = \frac{1}{4\pi\epsilon_0}.q\left[\frac{1}{r}\right]_\infty^x$$

$$V = \frac{1}{4\pi\epsilon_0}.q\left[\frac{1}{x}-\frac{1}{\infty}\right]$$

$$V_{out} = \frac{1}{4\pi\epsilon_0}.\frac{q}{x}$$

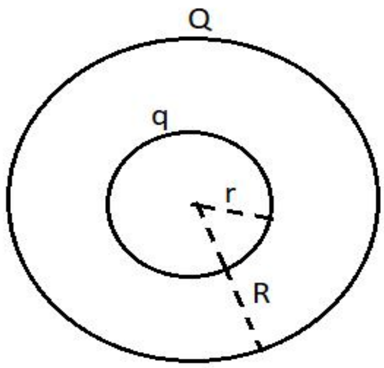
b. **At a point on the surface:**

$V_{surface} = \frac{1}{4\pi\epsilon_0}.\frac{q}{R}$

c. **At a point inside:**

$\frac{-dv}{dr} = 0$, V= constant, V = $V_{surface}$= $\frac{1}{4\pi\epsilon_0}.\frac{q}{R}$

48)If two concentric charged spherical shells are as shown in figure then find potentials on the surfaces of both spheres and also find potential difference between them.



Sol.

$$V_{in} = V_s + V_L$$

$$V_{in} = \frac{1}{4\pi\epsilon_0}.\frac{q}{r} + \frac{1}{4\pi\epsilon_0}.\frac{Q}{R}$$

$$V_{out} = V_s + V_L$$

$$V_{out} = \frac{1}{4\pi\epsilon_0}.\frac{q}{R} + \frac{1}{4\pi\epsilon_0}.\frac{Q}{R}$$

$$\Delta V = V_{in} - V_{out}$$

$$\Delta V = \frac{1}{4\pi\epsilon_0}.q\ [\frac{1}{r} - \frac{1}{R}]$$

Conclusion:

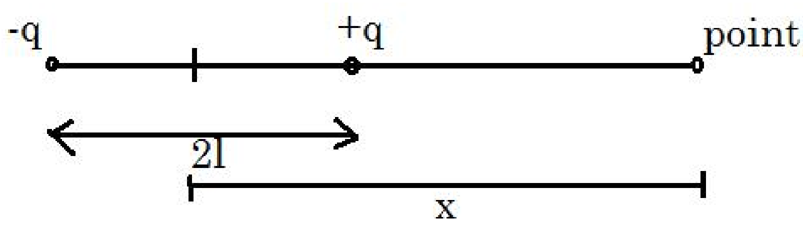
i) Potential difference is independent from the charge on the outer sphere.

ii) Potential of inner sphere is always higher than the potential of outer sphere.

iii) If we connect both the spheres by a conducting wire then the charge always flows from inner sphere to outer sphere.

Electric potential due to electric dipole:

Case-I: At a point on the axis of dipole



$$V = V_+ + V_-$$

$$V = \frac{Kq}{(x-l)} + \frac{K(-q)}{(x+l)}$$

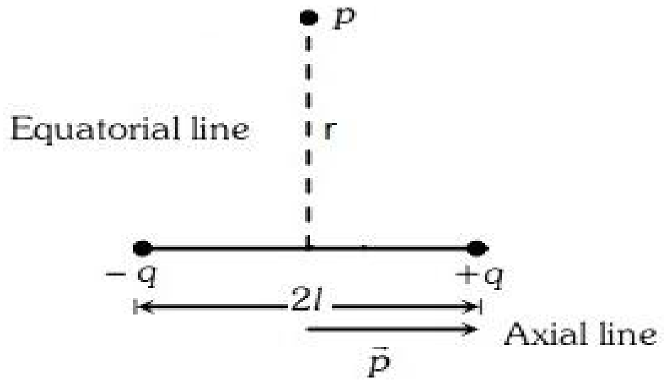
$$V = Kq\left[\frac{x+l-x+l}{x^2-l^2}\right]$$

$$V_{axis} = K\frac{p}{(x^2-l^2)}$$

For short dipole (x>>l)

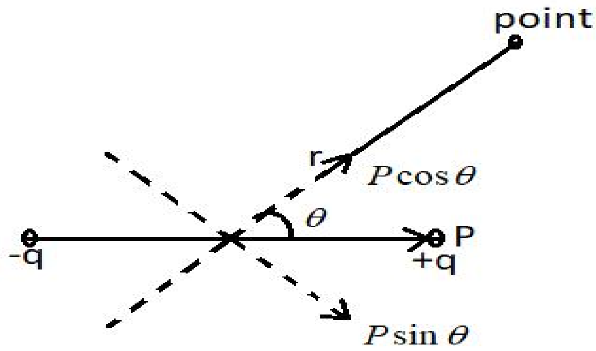
$$V_{axis} = k\frac{p}{x^2}$$

Case.II:- At a point on the equator of the dipole.



$$V_{equator} = 0$$

Case-III:- Electric potential at a general points due to an electric dipole:



$$V_{net} = \frac{kP\cos\theta}{x^2} + 0$$

$$\boldsymbol{V_{net} = \frac{kP\cos\theta}{x^2}}$$

From this last expression we can find potential at every point

Capacitor & Dielectric

Capacitor: Capacitor is an electrical device which stores electrical energy.

Note:- Net charge on a capacitor is always zero.

capacitor stores energy but not the charge.

Capacitance of a capacitor:

Capacitance represent amount of charge required to change the potential a capacitor (of a metal surface) by 1 volt.

$$c = \frac{q}{V}$$

$q = CV$, V= potential of the capacitor, q= charge of capacitor

SI unit of capacitor is Farad.

In more clear way,

$$C = \frac{q}{\Delta V}$$

$$q = C\Delta V$$

Here ΔV is change in potential due to charge given (q).

Higher the value of C represents that the system stores more electrical energy or we can say that the system requires more charge to change its potential.

49)Calculate capacitance of capacitor of which radius is equal to radius of earth.

Sol. Let we supply a charge q, then change in potential V of the sphere is given by:

$$V = \frac{1}{4\pi\epsilon_0}.\frac{q}{R}$$

But $C = \frac{q}{V}$

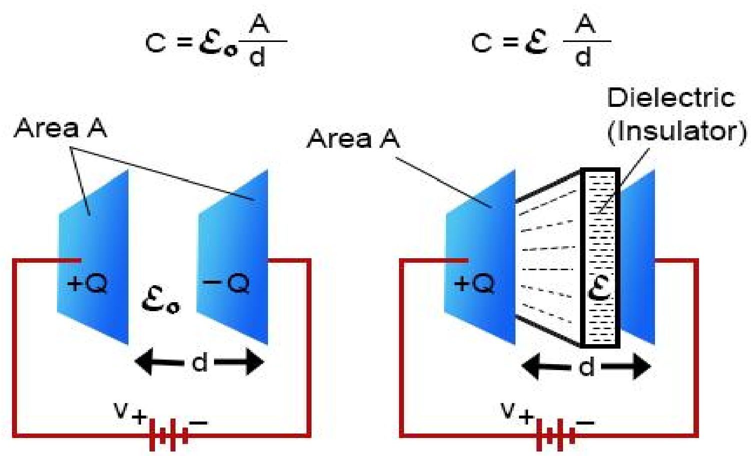
$$C = \frac{q}{\frac{1}{4\pi\epsilon_0}.\frac{q}{R}}$$

$C = 4\pi\epsilon_0 R$ Capacitance of spherical capacitor

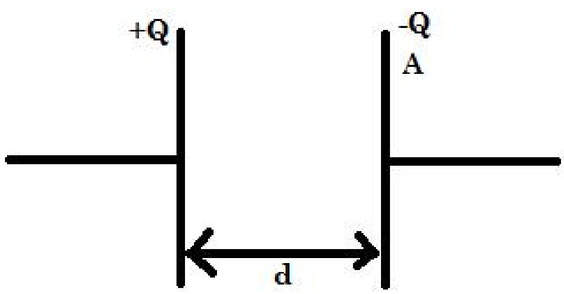
Note:- Capacitance is independent from the charge on capacitor. It depends on shape and size of capacitor and medium.

From the last equation it is clear that 1F is very large value of capacitance, practically we can't design such a large capacitor, hence we always use capacitors of capacitance in µF or even low.

Parallel plate capacitor:



Calculation of capacitance of parallel plate Capacitor:



Let charge given to a parallel plate capacitor is "Q" then net electric field between its both plate is:

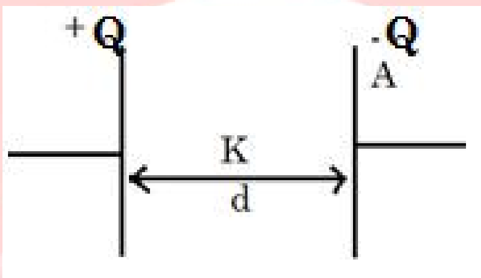
$$E = \frac{\sigma}{\epsilon_0}$$

$$\frac{\Delta V}{d} = \frac{Q/A}{\epsilon_0}$$

$$\frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}$$

$$C_0 = \frac{\epsilon_0 A}{d}$$

<u>Capacitance of parallel plate Capacitor completely filled with a dielectric:</u>



Let charge given to capacitor be "Q"

$$E_m = \frac{\sigma}{K\epsilon_0}$$

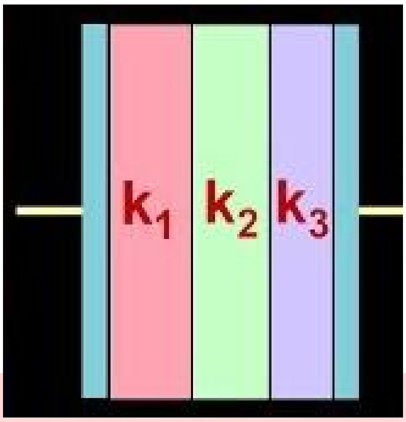
$$\frac{\Delta V}{d} = \frac{Q}{A\epsilon_0 K}$$

$$\frac{\theta}{\Delta V} = \frac{K\epsilon_0 A}{d}$$

$$C_m = \frac{K\epsilon_0 A}{d}$$

$$C_m = KC_0, \qquad C_m > C_0 \qquad as\ K \geq 1$$

Parallel plate Capacitor filled with different types of dielectric medium:



$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$$

$$\Delta V = E_1 t_1 + E_2 t_2 + E_3 t_3$$

$$\Delta V = \frac{\sigma}{K_1 \epsilon_0} t_1 + \frac{\sigma}{K_2 \epsilon_0} t_2 + \frac{\sigma}{K_3} \epsilon_0 t_3$$

$$\Delta V = \frac{Q}{A\epsilon_0} [\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3}]$$

$$\frac{Q}{\Delta V} = \frac{A\epsilon_0}{\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} \ldots.}$$

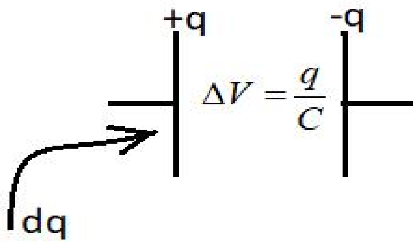
$$\frac{Q}{\Delta V} = C$$

$$C = \frac{A\epsilon_0}{\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} \ldots.}$$

$$C = \frac{A\epsilon_0}{d - t + \frac{t}{K}}$$

Energy stored in a parallel plate capacitor:

During the charging of a capacitor, work done by battery is stored in the form of electric potential energy in the capacitor.



Let at any instant charge on a capacitor is q, hence potential difference between its both plates is

$$\Delta V = \frac{q}{C}$$

At this instant work done by battery to supply a charge dq is given by

$$dW = dq\,(\Delta V).$$

$$dW = dq\,(\frac{q}{C}).$$

Net work doen by the battery is $W = \int_0^Q \frac{q}{C}\,(dq)$

$$W = \frac{1}{2C}\,.\,[q^2]_0^Q$$

$$W = \frac{1}{2}\,.\,\frac{Q^2}{C}$$

Because initial energy was zero hence this work done is stored in the form of potential energy in the capacitor. It means $U = \frac{1}{2}\,.\,\frac{Q^2}{C}$

This equation can be written as following also:

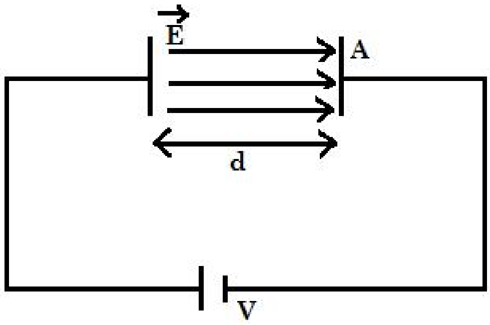
$$U = \frac{1}{2}\,.\,\frac{Q^2}{C}$$

$$U = \frac{1}{2}CV^2, \quad [Q = CV]$$

$$U = \frac{1}{2}QV \qquad , [C = \frac{Q}{V}]$$

Energy stored per unit volume in a capacitor or Energy density in electric field:-

Let a parallel plate capacitor as shown in figure:



$$U = \frac{1}{2} . CV^2$$

$$U = \frac{1}{2} \frac{\in_0 A}{d} (Ed)^2$$

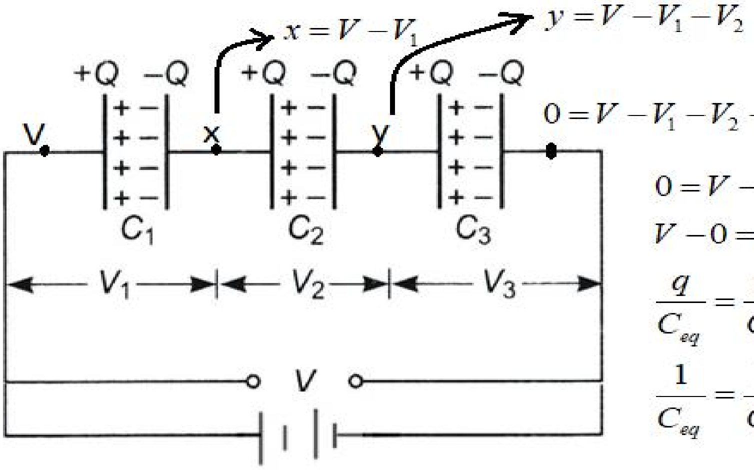
$$U = \frac{1}{2} \in_0 (Ad) E^2$$

$\frac{U}{Ad} = \frac{1}{2} \in_0 E^2$ U is total energy

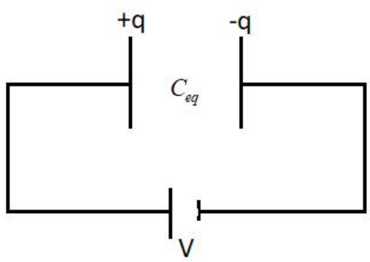
$u = \frac{1}{2} \in_0 E^2$ u is energy density

Combination of capacitors:-

Series combination:



$$0 = V - V_1 - V_2 - V_3$$
$$V - 0 = V_1 + V_2 + V_3$$
$$\frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



In series combination we can make following conclusions:

- charge on each capacitor is same and i.e. equal to the charge drawn from the cell.

V_1, V_2, V_3 are potential differences between the terminals of capacitor.

- Potential difference across each capacitor is different. In this case net supply voltage has been divided in each capacitor.

$$(V - x) + (x - y) + (y - 0) = V_1 + V_2 + V_3$$

$$V - 0 = V_1 + V_2 + V_3$$

$$V = V_1 + V_2 + V_3$$

- For only two capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{C_1 + C_2}{C_1 C_2}$$

$$\frac{1}{C_{eq}} = \frac{C_1 C_2}{C_1 + C_2}$$

- Voltage division rule in series combinaiton:

$$Q = CV$$

CV = Constant [Q is same for each capacitor]

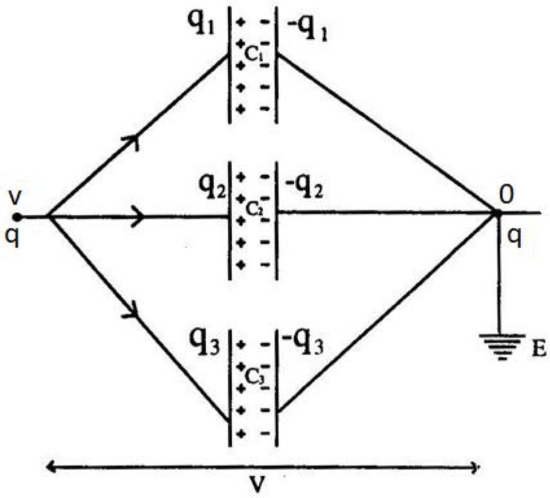
$$V = \frac{Constant}{C}$$

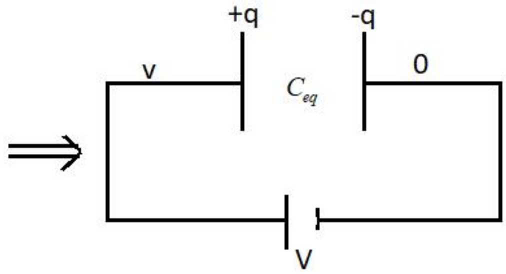
$$V \propto \frac{1}{C}$$

$$V_1 : V_2 : V_3 .... = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

- For only two capacitors in series $\frac{V_1}{V_2} = \frac{C_2}{C_1}$

Parallel Combination:-





$$q = q_1 + q_2 + q_3$$
$$C_{eq}V = C_1V + C_2V + C_3V$$
$$C_{eq} = C_1 + C_2 + C_3$$

Note:-

- Potential difference across each capacitor is same.
- Net charge supplied by the battery divides in each capacitor.

Charge division Rule in parallel combination:

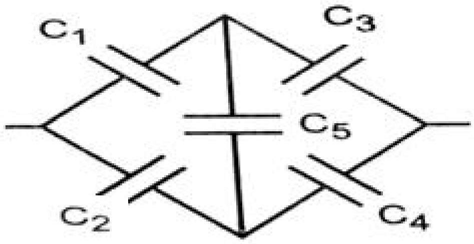
$$Q = CV$$

$$Q \propto C$$

$$Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$$

For only two capacitors in parallel combination $\frac{q_1}{q_2} = \frac{C_1}{C_2}$

Wheat stone bridge: A combination of 5 capacitors as shown in figure is called Wheatstone bridge.



Balanced wheat stone bridge: when potentials across the capacitors C_5 have same values then the bridge is balanced.

In this case charge flow through the capacitor C_5 is zero. $\Rightarrow Q = C_5(x - x)$

$Q = 0$. Hence the capacitor C_5 does not work.

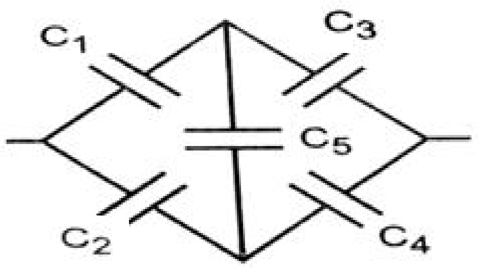
Condition of balance wheat stone bridge:

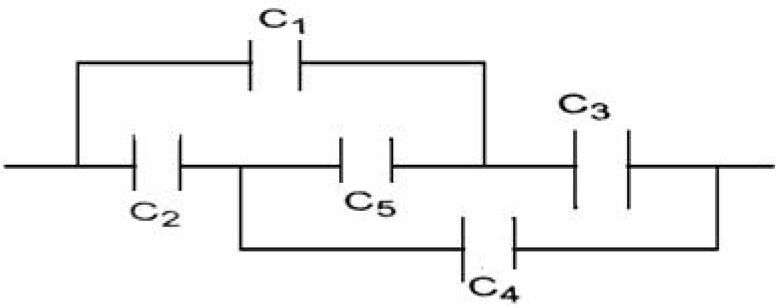
$Q_1 = C_1(V - x) = C_3(x - 0)$.......(1)

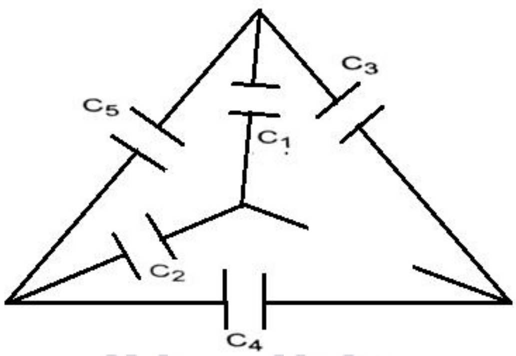
$Q_2 = C_2(V - x) = C_4(x - 0)$......(2)

$$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$

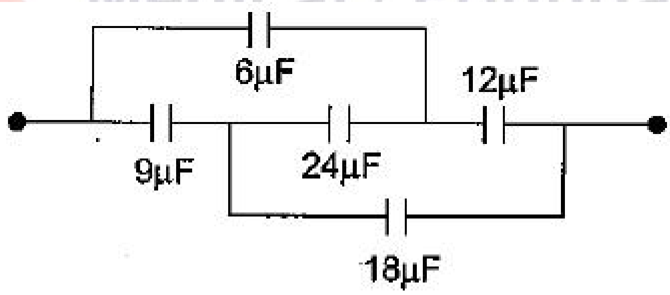
Different forms of wheat stone bridge:





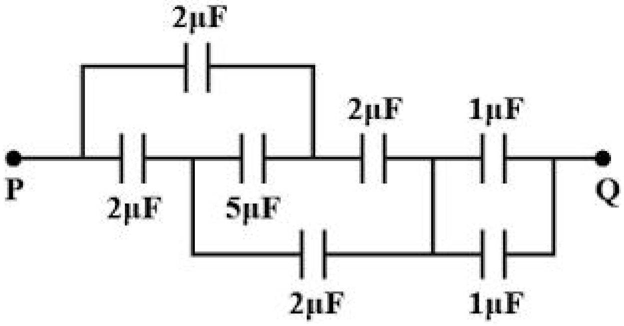


50)Find equivalent capacitance across the given terminals of the circuit.

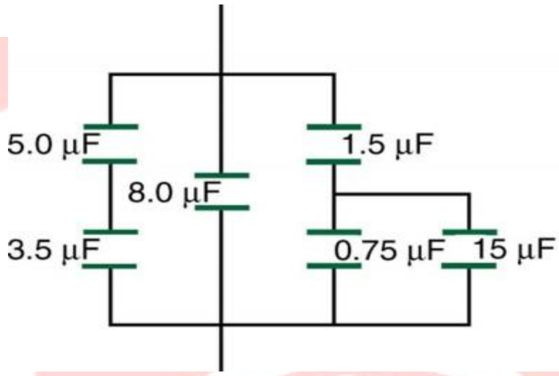


Ans: 10μF

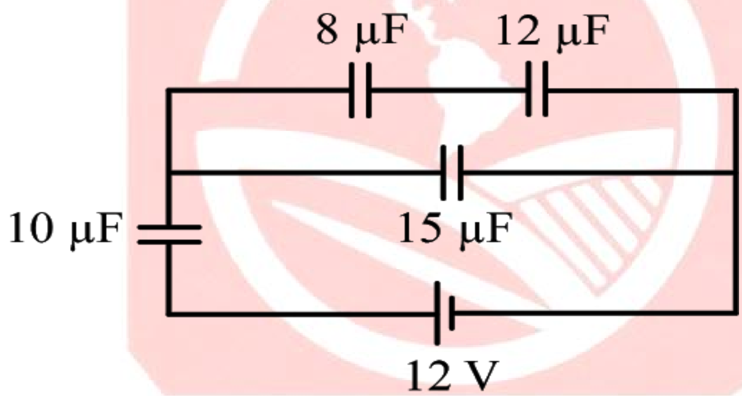
51) Find equivalent capacitance between the terminals P and Q.



52)Find equivalent capacitance across the given terminals

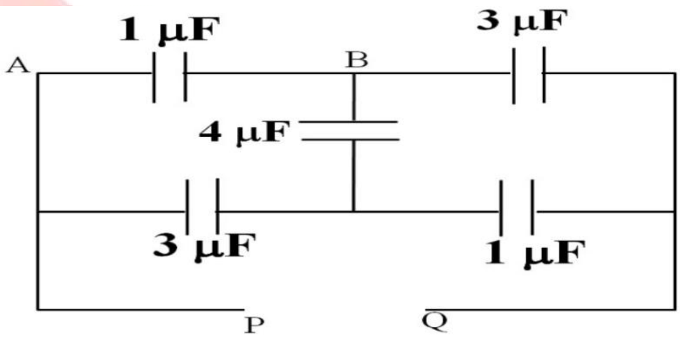


53)Find charge on each capacitor.



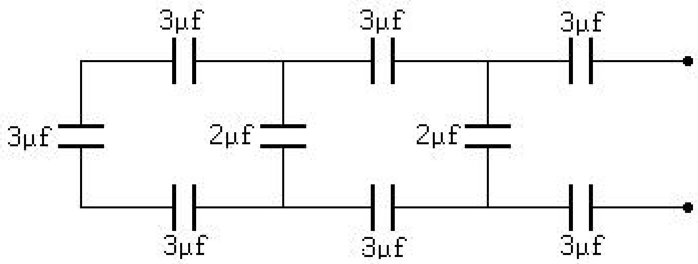
Unbalanced Wheatstone bridge:

54)Find equivalent capacitance across the given terminals.



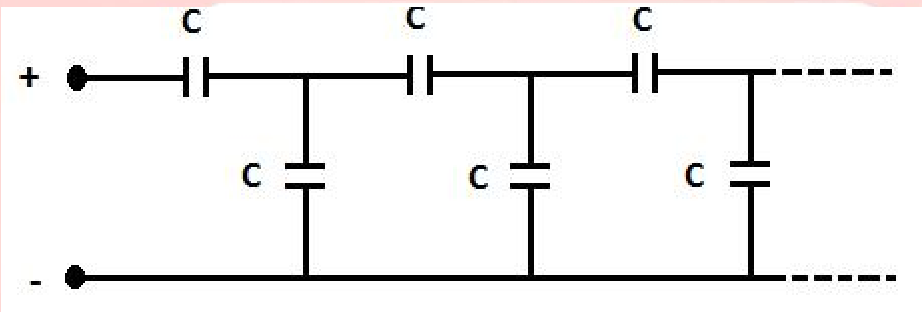
Ladder structure:

55)Find equivalent capacitance across the given terminals.



Infinite ladder structure:

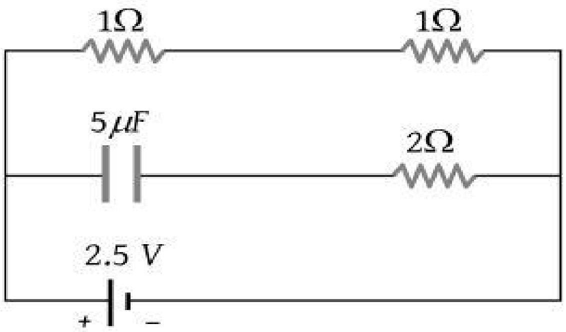
56) Find equivalent capacitance across the given terminals.



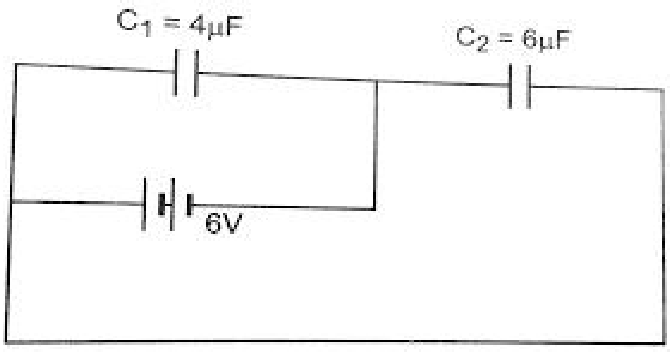
Note: We will discuss complex circuits of series and parallel in next chapter "current electricity"

Steady State of capacitor:- Capacitor is fully charged and current flowing through it is zero.

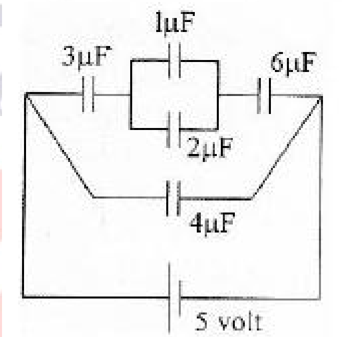
57) At steady state find current through each circuit element.



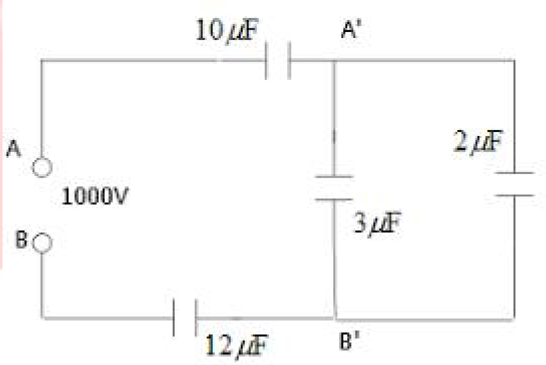
58) Find charge on each capacitor.



59) **Find charge on each capacitor**.

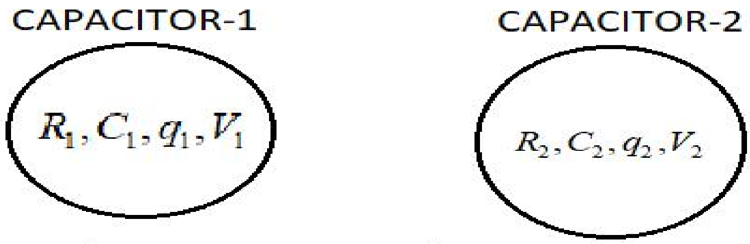


60) **Find charge on each capacitor.**

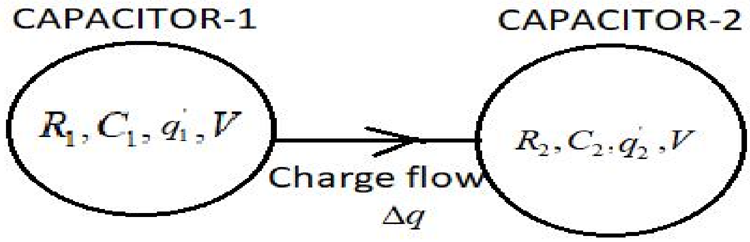


Redistribution of charges between two capacitors:

Iinitialy when capacitors are not connected



When capacitors are connected:



- **Calculation of common potential:**

$$q_1 + q_2 = q_1''' + q_2''$$

$$C_1V_1 + C_2V_2 = C_1V + C_2V$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$V = \frac{R_1V_1 + R_2V_2}{R_1 + R_2}$$

- **Calculation of charge transferred:**

$$\Delta q = q_1 - q_1'' \text{ or } q_2'' - q_2$$

$$\Delta q = C_1V_1 - C_2V$$

$$\Delta q = C_1V_1 - C_1\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)$$

$$\Delta q = \frac{C_1^2V_1 - C_1C_2V_1 - C_1^2V_1 - C_1C_2V_2}{C_1 + C_2}$$

$$\Delta q = \frac{C_1C_2(V_1 - V_2)}{(C_1 + C_2)}$$

- **Calculation of loss of energy**:

$$\Delta U = U_j - U_i$$

$$\Delta U = \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_1V^2\right) - \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right)$$

$$\Delta U = \frac{1}{2}\left[(C_1 + C_2)\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2 - C_1V_1^2 - C_2V_2^2\right]$$

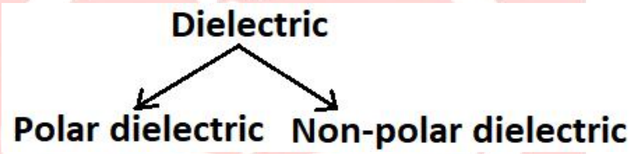
$$\Delta U = \frac{1}{2}\left[\frac{C_1^2V_1^2 + C_2^2V_2^2 + 2C_1C_2V_1V_2 - C_1^2V_1^2 + C_2^2V_2^2 - C_1C_2V_2^2 - C_2^2V_2^2}{C_1 + C_2}\right]$$

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{(C_1 + C_2)}\left[-V_1^2 - V_2^2 + 2V_1V_2\right]$$

$$\Delta U = -\frac{1}{2}\frac{C_1C_2}{(C_1 + C_2)}(V_1 - V_2)^2$$

Note:- Negative sign represents that there is energy loss in this process. In the form of heat.

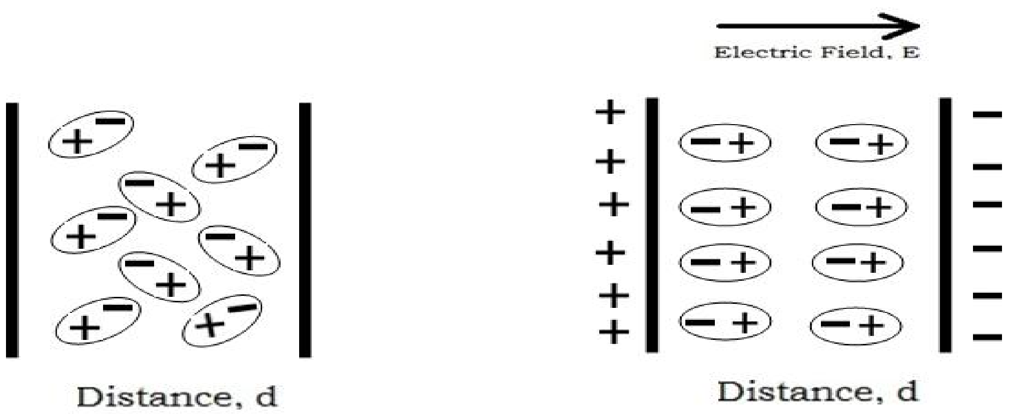
Dielectric Polarization:



Polar dielectric:

(1) Every molecule of polar dielectric always has a dipole moment but in the absence of external electric field entire substance has net dipole moment zero. (Due to random orientation of molecules)

(2) When we apply external electric field then the substance is polarized by the electric field.



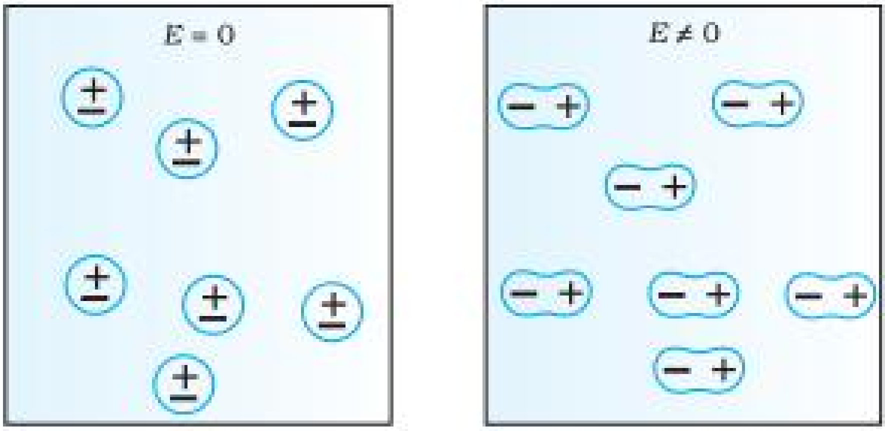
A. Polar molecules in dielectric without electric field

B. Polar molecules in the dielectric with the application of electric field

Non-polar dielectric:

(1) In the absence of external electric field every molecule has zero dipole moment [because centre of mass of net positive and net negative charge coincides].

(2) When we apply external electric field then each molecule gets polarised slightly due to result of which entire substance is polarised **in the direction of external electric field.**



(a) Non-polar molecules

Note:

1) Polar molecule: A molecule in which centres of mass of net positive and net negative charge do not coincide is called polar molecule.

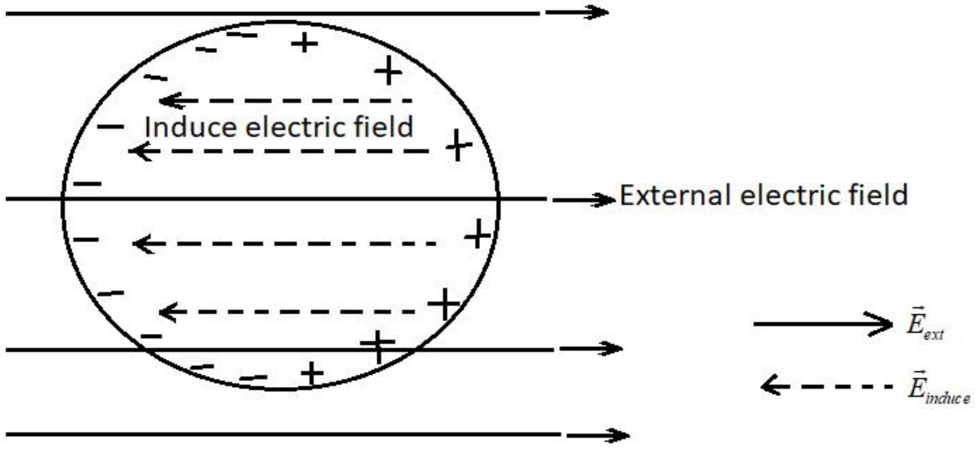
   Example: HCl, H_2O, NaCl . . . . . .etc.

2) Non-polar molecule: A molecule in which centres of mass of net positive and net negative charge coincide is called non-polar molecule.

   Example: N_2, O_2 . . . . …etc.

Conductor in external electric field: A substance in which there are most of the electrons free is called conductor.

➢ When we apply external electric field to a conductor then each electron experiences electric force due to result of which an induce electric field is prodeced inside the conductor in opposite direction of external electric field as shown in fig. and at equilibrium net electric field inside the conductor becomes zero.

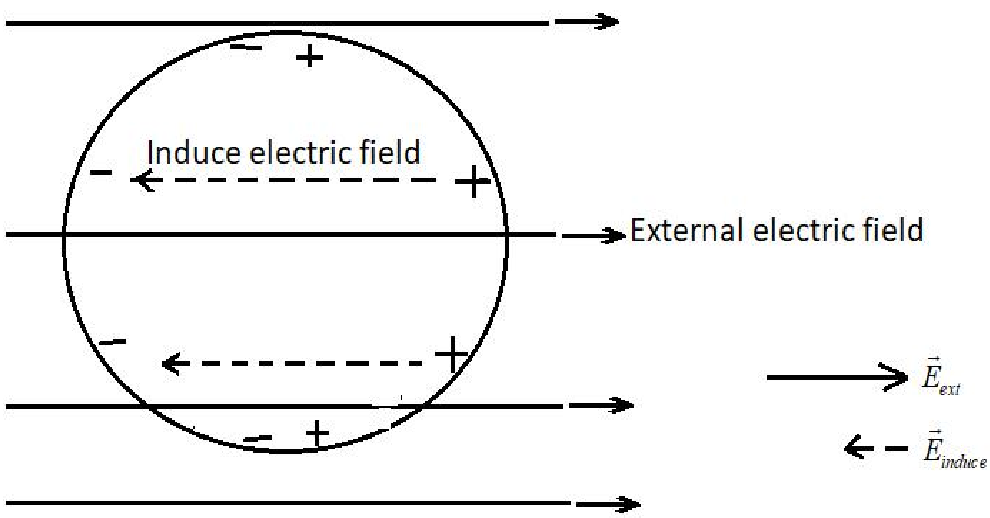


In this case $E_{ext}=E_{induce}$ hence

$$\vec{E}_{net} = \vec{E}_{ext} + \vec{E}_{induce}$$
$$E_{net} = E_{ext} - E_{induce}$$
$$E_{net} = 0$$

Dielectric in external electric field: A substance in which there are limited number of free electrons is called dielectric.

- When we apply external electric field to a dielectric then each electron experiences electric force due to result of which an induce electric field is prodeced inside the conductor in opposite direction of external electric field as shown in fig. and becouse of insufficient number of free electrons, net electric field inside the conductor can not be zero.



In this case $E_{ext} > E_{induce}$ hence net electric field can not be zero.

$$\vec{E}_{net} = \vec{E}_{ext} + \vec{E}_{induce}$$
$$E_{net} = E_{ext} - E_{induce}$$
$$E_{net} \neq 0$$

Breakdown electric field: The electric field applied to a conductor at which ionization of the conductor starts is called breakdown electric field.

In otherwords we can say that equivalent electric field of ionization potential is called breakdown electric field.

Note : Electric field inside a metal is inversaly proportional to dielectric constant of the metal.

$$\frac{E_1}{E_2} = \frac{K_2}{K_1}$$

Exercise

Coulomb force $\left(F = k\frac{q_1 q_2}{r^2}\right)$**, methods of charging & effect of medium:**

1. Two charges q_1 and q_2 are placed at a distance of 1m such that electrical repulsion between them is 54×10^{-3} N. If sum of both the charges is $5\mu C$ then find the values of individual charges.

   Ans. $2\mu C, 3\mu C$

2. Two charges q_1 and q_2 are placed at a distance of 2m such that electrical attraction between them is 27×10^{-3} N. If sum of both the charges is $1\mu C$ then find the values of individual charges.

   Ans. $4\mu C, -3\mu C$

3. Two small identical metallic spheres A and B each carrying a charge q repel each other with a force F. A third uncharged metallic sphere C of the same size is successively made to touch the

spheres A and B and then removed away. What is the new force of repulsion between A and B?

Ans. $(3/8)F$.

4. Two pieces of copper each of mass 10g are 10cm apart from each other. One electron per 1000 atoms is transferred from one piece of copper into the other. How much Colombian force will act between them after the transference of electrons? Atomic weight of copper is 63.5 g/mol, Avogadro's number $= 6\times10^{23}$/mol. Ans. $2.057\times10^{14}N$

5. Two identical conducting spheres having unequal charges of opposite signs, attract each other with a force of 0.108N when seperated by 0.5m. The spheres are connected by a conducting wire, which is then removed and thereafter repel each other with a force of 0.036N. What were the initial charges on the spheres?

Ans. $q_1 = \pm 3\mu C$ $q_2 = \mp 1\mu C$

6. Two identical metallic spheres having unequal opposite charges are placed at a distance $0.90m$ apart in air. After bringing them in contact with each other they are again placed at the same distance apart. Now the force of repulsion between them is $0.025N$. Calculate the final charge on each of them. Ans. $\pm1.5\mu C$.

7. Two identical point-charge Q, Q are kept at a distance 'r' apart. A third point-charge q is placed on the line joining the above two charges such that all the three charges are in equilibrium. What is the magnitude, sign and position of the third charge?

Ans. A negative charge of magnitude $Q/4$ should be placed mid-way between the charges $+Q$ and $+Q$. In this condition the net force on each charge will be zero. The equilibrium is unstable

Applications of Coulomb force:

Vector addition:

8. Two similarly and equally charged identical metal spheres A and B repel each other with a force of $2\times10^{-5}N$. A third identical uncharged sphere C is touched with A and then placed at the mid-point between A and B. calculate the net electric force on C. Ans. $2\times10^{-5}N$

9. Two equal point-charges $Q=+\sqrt{2}\mu C$ are placed at each of the two opposite corners of a square, and equal point-charges q at each of other two corners. What must be the value of q so that the resultant force on Q is zero? Ans. $=-0.5\mu C.$

10. Four point charges 2 μC, –5 μC, 2 μC, and –5 μC are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 μC placed at the center of the square?

11. Consider the charges q, q, and –q placed at the vertices of an equilateral triangle. What is the force on each charge?

12. Three charges (each $+q$) are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the center of triangle.

    (a) If $Q=-q$, will the charges at the corners move towards the center or fly away from it?

    (b) For what value of Q will all the four charges remain stationary?

    Ans. (a) Move towards the center (b) $Q=-q/\sqrt{3}$

13. Two electric charges +q and +2q are at a distance 'a' apart from each other in air. A third charge Q is to be placed along the same line in such a way that the net force acting at q and also at $2q$ is zero. Calculate the position of charge Q in terms of a.

    Ans. $\frac{a}{1+\sqrt{2}}$ $from\ the\ charge\ +q\ towards\ the\ chrage\ +2q$

Equilibrium problems:

14. Two point-charges $+4e$ and $+e$ are **fixed** at a distance 'a' apart.

Where should a third point-charge q be placed on the line joining the two charges so that **it** will be in equilibrium? Ans. Distance $2a/3$ from the charge $+4e$ in between the two charges.

(b) In which condition the equilibrium will be stable and in which unstable?

15. Two **free** point-charges +4e and +e are at a distance a apart.

(a)Where should a third point-charge q be placed between them such that the **entire system** be in equilibrium?

(b)What will be the magnitude and sign of q?

(c)what type of equilibrium will it be?

Ans. At $\frac{2a}{3}$ $from + 4e\ towards + e,\ q = -\frac{4e}{9},$ unstable.

16. Three point-charges of $+2\mu C, -3\mu C$ and $-3\mu C$ are kept in the vertices A, B and C respectively of an equilateral triangle of side $20cm$. What should be the sign and magnitude of a charge to be placed at the mid-point M of side BC so that the charge at A remains in equilibrium? Ans. $\frac{9\sqrt{3}}{4}\mu C.$

SHM Problems:

17. A particle of mass m and charge +q is located midway between two fixed charged particles each having a charge +q and at a distance 2L apart. Assuming that the middle charge moves along the line joining the fixed charges, then prove that its motion is SHM and also find its frequency of oscillation when it is displaced slightly and then release.

Ans. $\frac{q}{2\pi}\sqrt{\frac{1}{m\pi\varepsilon_0 L^3}}.$

18. A particle of mass m and charge -q is located midway between two fixed charged particles each having a charge +q and at a distance 2L apart. Assuming that the middle charge moves

perpendicular to the line joining the fixed charges, then prove that its motion is SHM and also find its frequency of oscillation when it is displaced slightly and then release.

Effect of medium:

19. Two identical charged spheres are suspended in air by strings of equal lengths and make an angle of $30°$ with each other. When suspended in a liquid of density $800 kg/m^3$, the angle remains the same. What is the dielectric constant of the liquid? (Density of the material of the spheres $= 1600 kg/m^3$.) Ans. K = 2

20. Two small insulated copper spheres A and B of same size have their centers 50 cm apart in air. (a)Find the mutual force of repulsion, if charge on each is $+6.5\times10^{-7} C$.

(b)What would be the force of repulsion if (i) the charge on each sphere be doubled and the distance between them be halved (ii) both spheres A and B be placed in water (K=81) (iii) a third identical uncharged sphere C be touched with A and then with B and finally removed away.

Ans. (a) $1.5\times10^{-2} N$ (repulsive). (b) (i) $1.5\times10^{-2} N$ (ii) $1.85\times10^{-4} N$ (iii) $5.7\times10^{-3} N$

Circular motion problem:

21. A ball of mass 10^{-2} kg and having charge $+3\times10^{-6} C$ is tied at one end of a 1 m long thread. The other end of the thread is fixed and a charge $-3\times 10^{-6} C$ is placed at this end. The ball can move in the circular orbit of radius 1 m in the vertical plane. Initially, the ball is at the bottom. Find the minimum initial horizontal velocity of the ball so that it will be able to complete the full circle. Ans. $= 7.62 m/s$.

Force due to electric field $\vec{F} = q\vec{E}$:

22. An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude $2.0\times10^{4}\ NC^{-1}$.The direction of the field is reversed and a proton falls through the same distance. Calculate the time of fall in each case. Ignoring gravity.

23. A charged dust particle of radius 5×10^{-7} m is located in a horizontal electric field having an intensity of 6.28×10^{5} V/m. The surrounding medium is air with coefficient of viscosity $\eta = 1.6\times10^{-5}\ N-s/m^2$. If this particle moves with a uniform horizontal speed 0.02m/s, find the number of electrons on it.

Ans. = 30.

24. Find the magnitude and direction of an electric field that will balance an alpha particle. The mass and charge of a proton are $1.67\times10^{-27}\ kg$ and $+1.6\times10^{-19}\ C$. Take g = 9.8 N/kg. Ans. $=2.0\times10^{-7}\ N/C$.

25. *A drop having a mass of 4.8×10^{-10} g and a charge of $2.4\times10^{-18}\ C$ is suspended between two charged horizontal plates at a distance 1.0 cm apart.

(i)Find electric field & potential difference between the plates.

(ii) If polarity of the plates be changed then calculate the instantaneous acceleration of the drop.

Ans. $19.6 m/s^2$

26. An oil drop having 12 excess electrons is held stationary under a uniform electric field of $2.55\times10^{4}\ NC^{-1}$.The density of the oil is $1.26 gcm^{-3}$. Estimate the radius of the drop.

Ans. = 9.81×10^{-4} mm.

27. *Two plane parallel conducting plates 1.5×10^{-2} m apart are held horizontally one above the other in air. The upper plate maintained at a positive potential of 1.5kV while the other plate is earthed.

(i)Calculate the number of electrons which must be attached to a small oil drop of mass 4.9×10^{-15} kg held stationary between the plates, assuming that the density of air is negligible in comparison with that of oil.

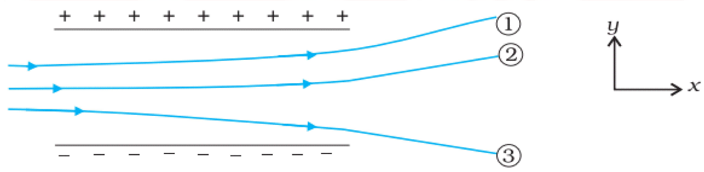
(ii) If the potential of the upper plate is suddenly changed to -1.5kV, what is the initial acceleration of the charge drop?

(iii) Also obtain the terminal velocity of the drop if its radius is 5.0×10^{-6} m and the coefficient of viscosity of air is $1.8\times10^{-5} Ns/m^2$

Ans. (ii) $19.6 m/s^2$, (iii) 5.7×10^{-5} m/s

28. A particle of mass m and charge q is thrown in the vertical direction with a velocity υ_0 in a uniform horizontal electric field E. Assuming the gravity force to be negligible, derive an equation of the path followed by the particle. Ans. $x=\frac{qE}{2m\upsilon_0^2}y^2$

29. Figure shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?

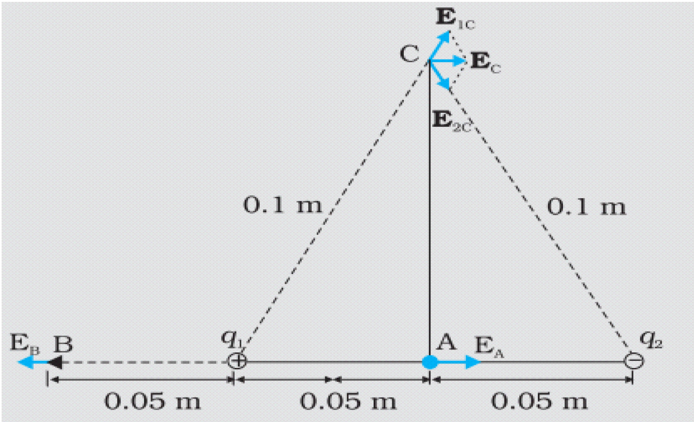


Electric field due to point charge & its applications:

30. An electron is separated from a proton by a distance of $0.53\,\mathring{A}$. Calculate the electric field at the location of the electron. Ans. $=5.13\times10^{11} NC^{-1}$.

31. Charges of $+1.2\times10^{-8}C$ and $-1.6\times10^{-8}C$ are placed at two points A and B respectively, distant 5 cm from each other. Compute the electric field at a point C distant 3 cm from A and 4 cm from B. Ans. $=1.5\times10^{5} N/C$

32. Two point-charges of $+10^{-8}C$ and $-10^{-8}C$ are placed $0.1m$ apart as shown. Calculate the electric fields at points A ,B and C.



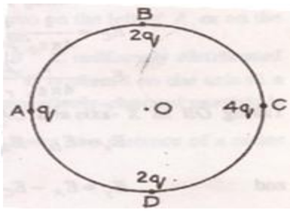
Ans. (i) $= 7.2\times10^4\,NC^{-1}$, $\vec{E}_A$ is directed towards right. (ii) $= 3.2\times10^4\,NC^{-1}$, $\vec{E}_B$ is directed towards left.

(iii) $= 9.0\times10^3\,NC^{-1}$, $\vec{E}_C$ is directed towards right.

33. Four point-charge q, 2q, 4q and 2q are placed at four points A, B, C and D respectively (as shown) on the circumference of a circle of radius $R = 2.0m$. The value of q is $4.0\,\mu C$. The lines AC and BD are perpendicular to each other.

(i)Write the formula for the electric field at the centre O of the circle due to each of these charges.

(ii)Obtain the magnitude and direction of the electric field at O, due to all four charges combined.



Ans. $2.7\times10^4\,NC^{-1}$, towards A.

34. Four point-charges each of $q = 0.02\,\mu C$ are placed at the corners of a square of side 2cm. Find the magnitude and direction of the electric field at the centre O of the square.

35. Two point-charge of $5\times10^{-19}\,C$ and $20\times10^{-19}\,C$ are separated by a distance of 2 m. At which point on the line joining them, the electric field is zero? Ans. The field is zero at a distance 2/3 meter.

Electric field due to a charged rod:

36. Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law.

Electric field due to a charged ring:

37. A charge of $4\times10^{-9}C$ is uniformly distributed over the surface of a ring-shaped conductor of radius 0.3 m. Calculate the intensity of the electric field at a point of the axis of the ring at a distance of 0.4 m and specify its direction. What is intensity at the center of the ring?

Ans. $=115.2Vm^{-1}$, zero

38. A thin stationary ring of radius L m has a positive charge of +q uniformly distributed over it. A particle of mass m and having a negative charge of -q is placed at the center of the ring. Show that the motion of the negatively-charged particle is approximately simple harmonic. Calculate the time-period of oscillation.

Electric dipole:

39. Three charges +q, -2q and +q are located at the vertices of an equilateral triangle of side $2l$ .What is the equivalent dipole moment of the arrangement?

Ans. $2\sqrt{3}ql$.

Electric field due to dipole:

40. Two point-charges $\pm10\mu C$ are placed 5.00 mm apart, forming an electric dipole. Compute electric field at a point on the axis of dipole 15 cm away from the center, and on a line passing through the center and normal to the axis of the dipole.

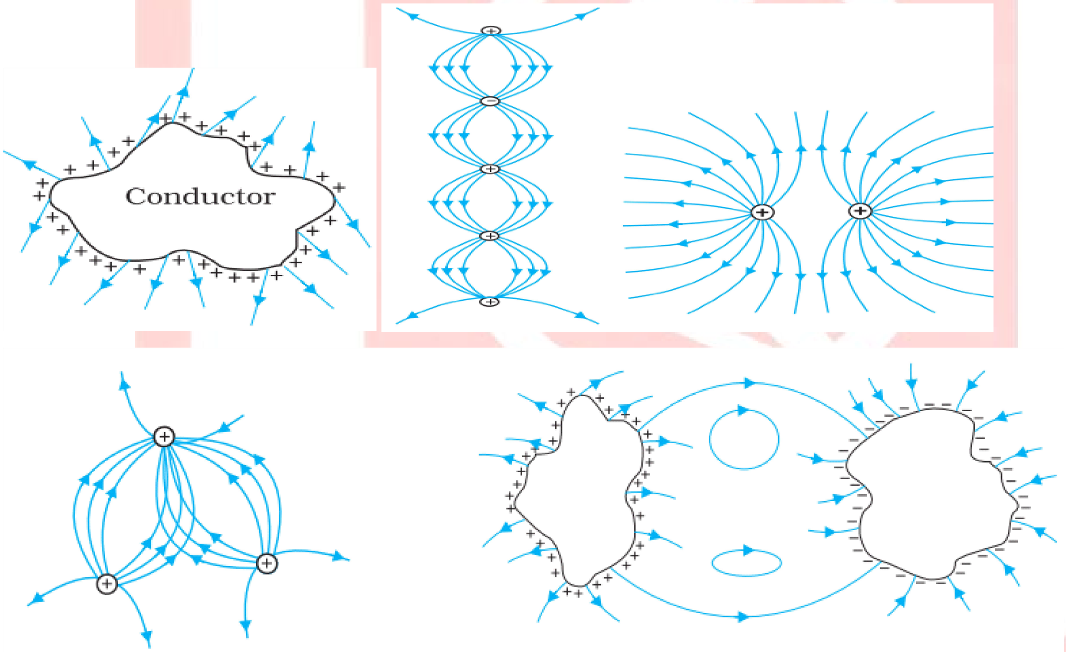
Ans. $2.66\times10^{5}NC^{-1}, 1.33\times10^{5}NC^{-1}$.

Force and torque on a dipole:

41. An electric dipole of moment 4×10^{-9} C m is aligned at $30°$ with the direction of a uniform electric field E $=5\times10^{4}\,NC^{-1}$. What is the magnitude of the force and the torque acting on the dipole?

42. *In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is however not constant but increases uniformly along the positive z-direction at the rate of $10^{5}\,N/C$ per meter. What are the force and torque experienced by a system having a total dipole moment equal to $10^{-7}\,Cm$ in the negative z-direction?

43. Which among the curves shown in Fig. cannot possibly represent electrostatic field lines?



Electric flux $\phi=\int\vec{E}.d\vec{A}$ **and Gauss theorem** $\oint\vec{E}.d\vec{A}=\frac{q_{in}}{\varepsilon_0}$**:**

44. A point-charge produces an electric flux of $-1.0\times10^{3}\,Nm^{2}C^{-1}$ which passes through a Gaussian sphere of radius 10 cm centered on the charge. Compute the point-charge. If the radius of the

sphere were doubled, how much flux would pass through its surface? Ans. $-8.85nC$.

45. A point-charge of $2.0\mu C$ is at the center of a cubic Gaussian surface 9.0 cm on edge.

(i)What is the net electric flux through the whole surface?

(ii)Through one face of the cube

(iii) What if the charge is not at the centre? Ans. $2.26\times10^5\,Nm^2C^{-1}$, $4.65\times10^6\,NC^{-1}$, Remain same

46. A charge of $17.7\times10^{-4}C$ is distributed uniformly over a large sheet of area $200m^2$. Calculate the electric field intensity at a distance 20 cm from it in air. Ans. $5\times10^5\,NC^{-1}$.

47. An electric flux of $-6.0\times10^3\,Nm^2/C$ passes normally through a spherical Gaussian surface of radius 10 cm, due to a point-charge placed at its center. What is the charge enclosed by the Gaussian surface? If the radius of the Gaussian surface is doubled, how much flux would pass through the surface? Ans. $-53.1nC$, $-6.0\times10^3\,Nm^2C^{-1}$.

48. *A thin spherical shell of metal has a radius of 0.25 meter and carries a charge of 0.2 micro-coulomb. Calculate the electric intensity at a point (i) inside the shell, (ii) just outside the shell and (iii) 3.0 meter from the center of the shell. Ans. (i) zero. (ii) $2.88\times10^4\,NC^{-1}$ (iii) $200NC^{-1}$.

49. **From what distance should a 100-eV electron be fired towards a large metal plate having a surface charge density of $-2.0\times10^{-6}\,Cm^{-2}$, so that is ‘just; fails to strike the plate?

Ans. 0.4425 mm.

50. *There are two large parallel metallic plates P_1 and P_2 carrying surface charge densities σ_1 and $\sigma_2(\sigma_1>\sigma_2)$ respectively, placed at a distance d apart in vacuum. Determine the work done by the electric field in moving a point-charge q from P_1 to P_2 along a line of length

a(a>d) making an angle of $\pi/4$ with the normal to the plate. Ans. $\frac{q(\sigma_1-\sigma_2)a}{\sqrt{2}\varepsilon_0}$.

51. If the electric field near the earth's surface be 300 V m^{-1} directed downwards, what is the surface density of charge on earth's surface? Ans. $2.65\times10^{-9}Cm^{-2}$.

Electric potential and work done $W_{ext}=q\Delta V$:

52. Two point-charges of $3\times10^{-8}C$ and $-2\times10^{-8}C$ are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero?

Ans. The potential is zero at a distance of 9 cm from the charge $3\times10^{-8}C$.

53. Find the potential at the center of a square of side $\sqrt{2}$ m which carries at its four corners charges $+2\times10^{-9}C$, $+1\times10^{-9}C$, $-2\times10^{-9}C$ and $+3\times10^{-9}C$. Ans. 36 V.

54. A regular hexagon of side 10 cm has a charge 5 μ C at each of its vertices. Compute electric potential at the center of the hexagon. Ans. 2.7×10^{6} V.

55. A cube of side b has a charge q at each of its vertices. Compute the electric potential and field due to this arrangement of charges at the center of the cube. Ans. The net electric field at the center is zero.

56. Four charges +q,+q,-q,-q are placed at the corners A, B, C, D respectively of a square of a side a. (i)Calculate electric potential and electric field at the center O of the square.

(ii) If E and F are mid-points of the side BC and CD respectively, then what will be the work done in carrying an electron (charge e) from O to E and from O to F ?

Ans. (i) $\frac{\sqrt{2}q}{\pi\varepsilon_0 a^2}$, acting vertically downward , $\frac{qe}{\pi\varepsilon_0 a}\left(1-\frac{1}{\sqrt{5}}\right)$.

57. *Two points A and B are 2 cm apart and a uniform electric field $\vec{E}$ acts along the straight-line AB directed from A to B with E = $200NC^{-1}$. A particle of charge $+10^{-6}$ C is taken from A to B along AB. Calculate

(a) the force on the charge

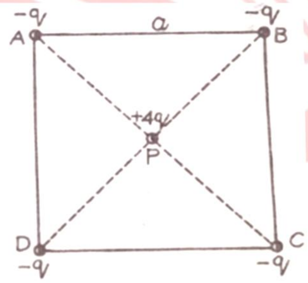
(b) the potential difference ($V_A - V_B$) and

(c) the work done on the charge by $\vec{E}$.

Ans. (a) 2×10^{-4} N along AB. (b) 4V. (c) 4×10^{-6} J.

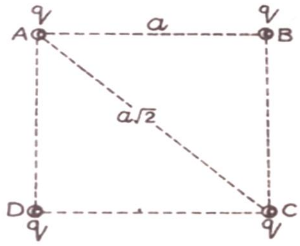
58. Calculate the potential at a point P due to a charge of 4×10^{-7} C located 9 cm away. Hence obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought? Ans. 4×10^{4}, 8×10^{-5} J, no

59. For the given system of charges given below, find the work required to remove the charge +4q from point P to infinity.



Ans. $+\frac{1}{4\pi\varepsilon_0}\frac{16\sqrt{2}q^2}{a}$.

60. Four equal charges q are brought from infinity to the four corners A, B, C, D of a square of side a, as shown. Compute the work required in bringing (i) the first charge, (ii) the second charge, (iii) the third charge, (iv) the fourth charge, (v) What will be the electrostatic potential energy of the whole system?



Ans. (i) 0, (ii) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}$, (iii) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(\frac{1}{\sqrt{2}}+1\right)$, (iv) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(2+\frac{1}{\sqrt{2}}\right)$, (v) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(4+\sqrt{2}\right)$

61. *At a point the electric field intensity and potential due to a point-charge are $32NC^{-1}$ and $16JC^{-1}$ respectively. Calculate:

(i)the distance of the charge from the point of observation,

(ii)magnitude of the charge. Ans. (i) 0.5 meter, (ii) 0.89 C.

62. Two point-charges of $0.12\mu C$ and $-0.06\mu C$ are situated at a distance of 3.0 meter from each other. Calculate the electric field and potential at a point mid-way between them. How much work will be done in bringing a charge of $0.2\mu C$ from infinity to the mid-point P? Ans. 720 NC^{-1}, 360 V.

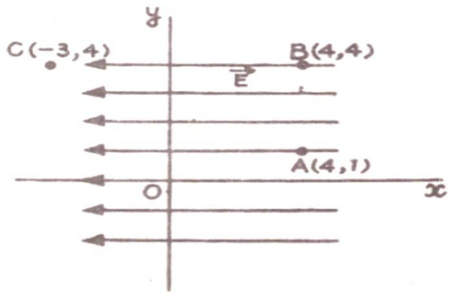
63. #*Two tiny spheres carrying charges 1.5μ C and 2.5μ C are placed 30 cm apart. Find electric potential at the mid-point P of the line joining the charges and at a point Q 10 cm from this mid-point in a plane normal to the line and passing through the mid-point.

Ans. 2.0×10^5 V.

Relation between $\vec{E}$ and V$\left(\vec{E}=-\frac{dV}{dr}\right)$:

64. An infinite plane sheet of charge density $10^{-8}Cm^{-1}$ is held in air. In the vicinity of the sheet, how far apart are two equipotential surfaces whose p.d. is 5 V ? Ans. 8.85 mm.

65. A uniform electric field $\vec{E}$ of 300 NC^{-1} is directed along $-x$ axis. A, B and C are three points in the field, having x and y coordinates (in meter) as shown. Calculate the potential differences between the points (i) A and B (ii) B and C and (iii) C and A.



66. #When the intensity of electric field becomes 3×10^{6} Vm^{-1} in air, then the insulation of air is broken down. How much maximum charge can be accumulated on a metallic sphere of diameter 60 cm? What will be the breakdown potential of the sphere? Given: $1/4\pi\varepsilon_0 = 9.0\times10^{9}\,Nm^2C^{-2}$. Ans. 9×10^{5} V.

Electric potential due to a ring:

67. *Two identical thin rings, each of radius R meter, are coaxially placed at a distance R meter apart. If Q_1 and Q_2 be the charges (in coulomb), spread uniformly on the two rings, then what will be the work required to move a charge q from the center of one ring to that of the other ?

Ans. $\dfrac{q(Q_1-Q_2)(\sqrt{2}-1)}{4\pi\varepsilon_0(\sqrt{2}R)}$.

#Electric potential due to a sphere:

68. Compute electric potential and potential gradient at a distance of 1.0×10^{-12} m from the center of a gold nucleus. Hence find electric field at that point. The atomic number of gold is 79.

Ans. $1.1\times10^{17}\,Vm^{-1}$ (or NC^{-1}).

69. A point charge A of 5×10^{-9} C is placed in air. Calculate the work done while

(i)a point-charge B of 3×10^{-9} C completes one revolution in a circle around the charge A. The radius of the circle is 6×10^{-2} m.

(ii)The charge B is brought towards charge A from 6 cm to 5 cm. Ans. 4.5×10^{-7} J.

70. The diameter of a hollow metallic sphere is 60 cm and the sphere carries a charge of 500 μ C. Calculate the electric field and potential (i) at a distance of 100 cm from the center of the sphere, (ii) at the surface of the sphere and (iii) at a distance of 10 cm from the center of the sphere. Ans. (i) 4.5×10^{6} V, (ii) 1.5×10^{7} V, (iii) 1.5×10^{7} V.

71. A spherical drop of water carrying a charge of 3.0×10^{-10} C has a potential of 500 V at its surface. What is the radius of the drop? If two such drops combine to form a single drop, what is the potential at the surface of the new drop so formed? Given: $2^{2/3}$ = 1.59.

Ans. 5.4 mm, 795 V.

72. *A drop of water of mass 18×10^{-3} g falls away from the bottom of a charged conducting sphere of radius 20 cm, carrying with it charged of 10^{-9} C and leaving on the sphere a uniformly distributed charge of 2.5×10^{-6} C. What is the speed of the drop after it has fallen 30 cm?

Ans. 3.66 m/s.

73. A charged conducting sphere is located inside a bigger charged conducting spherical shell. Prove that the potential of the inner sphere is higher than the potential of the outer shell.

Note: *If the inner sphere be connected to the outer shell by a wire, charge would always flow from the inner sphere to the outer shell, irrespective of the magnitude and sign of the charge on the shell.*

74. *The radii of two concentric spherical conducting shells are r_1 and $r_2 (> r_1)$. The charge on the outer shell is q. What is the charge on the inner shell which is connected to the earth? Ans. $-q(r_1/r_2)$.

75. *A charge Q is distributed over two concentric hollow spheres of radii r and R(>r) such that the surface densities are equal. Find the potential at the common centre.

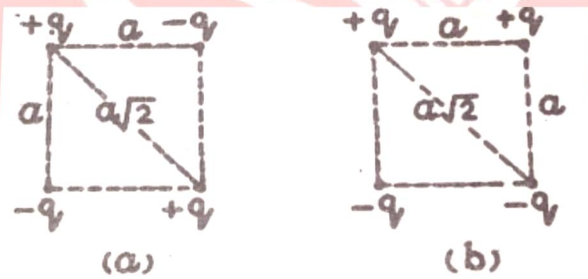
Ans. $\frac{Q}{4\pi\varepsilon_0}\left(\frac{R+r}{r^2+R^2}\right)$.

76. *Three concentric spherical metallic shells A, B and C of radii a, b and c(a<b<c) have surface charge densities $\sigma, -\sigma$ and σ respectively. (i) Find the potentials of the three shells A, B and C. (ii) If the shells A and C are at the same potential, obtain the relation between the radii a, b and c. Ans. a + b = c

Electrostatics potential energy:

77. (i)Calculate the work required to assemble each of the systems of charges (which is same as the electrostatic potential energy of the system) shown in Fig. (a) and (b).

(ii) #A charge q_0 is brought from infinity to the centre of the square, the four charges being held fixed at its corners. How much extra work is needed to do this. Ans. V = 0, W = 0



78. #A point charge q moves from point P to point S along the path PQRS in a uniform electric field E pointing parallel to the positive direction of the X-axis. The coordinates of the points P,Q,R and S are (a, b, 0), (2a, 0, 0)(a, -b, 0) and (0, 0, 0) respectively. Find the work done by the field in the process. Ans. $-qEa$

79. Determine the electrostatic potential energy of a system consisting of two charges $7\mu C$ and $-2\mu C$ separated by a distance of 18 cm. How much work is required to separate the two charges infinitely away from each other?

Ans. -0.7 J, 0 .7 J work will be done by some external agent in separating the charges (without any kinetic energy) up to infinity.

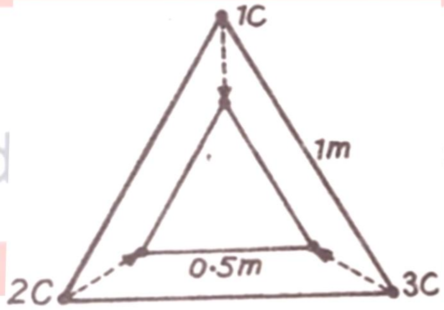
80. *Three point charges q, 2q and 8q coulomb are to be placed on a 9 cm long straight line.

(i)Find the position where the charges should be placed such that the potential energy of this system is minimum.

(ii)In this situation, what will be the electric field at the position of the charge q due to the other two charges ?

Ans. The charge q should be placed in between the charges 8q and 2q, at a distance of 6 cm ($=6\times10^{-2}$ m) from the charge 8q.

81. Three point-charges of 1 C, 2 C and 3 C are placed at the corners of an equilateral triangle of side 1 m. Calculate the work required to move these charges to the corners of a smaller equilateral triangle of side 0.5 m, as shown. (given: $\varepsilon_0^{-1} = 36\pi\times10^9 V-m/A-s$)

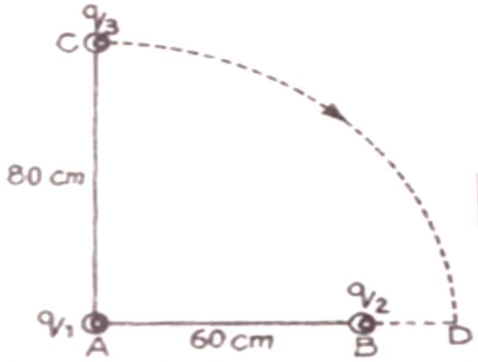


Ans. 99×10^9 J.

82. Two positive point-charges $q_1 = 0.2\mu C$ and $q_2 = 0.01\mu C$ are placed 10 cm apart in air. Compute the work required in (a) bringing the charges closer to 5 cm, (b) separating them to 15 cm and (c)removing q_2 from initial separation of 10 cm to infinity.

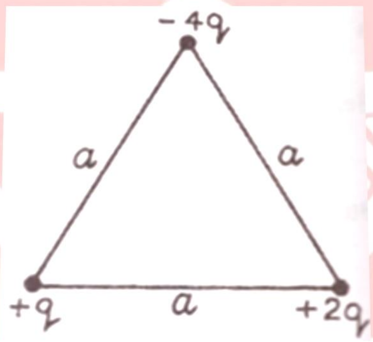
Ans. (a) 1.8×10^{-4} J, (b) -0.6×10^{-4} J, (c) -1.8×10^{-4} J.

83. *Two charges $q_1 = +2\times10^{-8}$ C and $q_2 = -0.4\times10^{-8}$ C are placed 60 cm apart, as shown. A third charge $q_3 = +0.2\times10^{-8}$ C is moved along the arc of a circle of radius 80 cm from C to D. Compute the percentage change in the potential energy of q_3.



Ans. Energy decreases 76%.

84. Three charges are arranged as shown. What is the electric potential energy of the system ? q= 150 nC (nano-coulomb) and a = 0.12m.



Ans. $-1.7\times10^{-2} J = -17 mJ$.

85. *Three charges of 0.1 C each are placed at the corners of an equilateral triangle of side 1.0 m. If the energy is supplied to the system at the rate of 1.0 kW, how much time would be required to move one of the charges on to the mid-point of the line joining the other two? $\left(\frac{1}{4\pi\varepsilon_0} = 9.0\times10^9 \frac{V-m}{A-s}\right)$ Ans. 1.8×10^5 s.

86. A neutral hydrogen molecule has two protons and two electrons. If one of the electron is removed, we get a hydrogen molecular ion (H_2^+). In the ground state of a H_2^+, the two protons are separated by roughly $1.5\,\mathring{A}$ and the electron is roughly $1.0\,\mathring{A}$ from each proton. Estimate the

potential energy of the system. Ans. -19.2eV.

87. In a hydrogen atom, the electron and proton are bound together at a distance of about $0.53\overset{\circ}{A}$.

(a) Estimate the potential energy of the system in eV, assuming zero potential energy at infinite separation between the electron and the proton.

(b) Find the minimum work to be done to free the electron, if the kinetic energy of the electron in its orbit is half the magnitude of the potential energy obtain in (a).

(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at $1.06\overset{\circ}{A}$ separation ?

Ans. (a) -27.2 eV, (b) 13.6 eV, (c) -13.6 eV (no work will be required to free the electron in this case).

Maximum kinetic energy of a charge particle accelerated by a potential difference $K_{\max} = q\Delta V$:

88. Obtain the energy in joule acquired by an electron beam when accelerated through a p.d. of 2000 V. How much speed will the electron gain ? Ans. 3.2×10^{-16} joule, $\frac{8}{3}\times10^{7}$ meter/second.

89. A proton moves with a speed of 7.45×10^{5} m/s directly towards a free proton originally at rest. Find the distance of closest approach for the two protons. Ans. 1.0×10^{-12} m.

90. What potential difference must be applied to produce an electric field that can accelerate an electron to one-tenth velocity of light ? The mass and charge of electron are 9.0×10^{-31} kg and 1.6×10^{-19} and velocity of light is $3.0\times10^{8}\,ms^{-1}$ Ans. 2531 V.

Electric dipole:

91. Two point-charges +2e and -2e are situated at a distance of $2.4\overset{\circ}{A}$ from each other and constitute an electric dipole. This dipole is placed in a uniform electric field of $4.0\times10^{5}Vm^{-1}$. Calculate : (i) electric dipole moment, (ii) potential energy of the dipole in equilibrium position, (iii) work done in rotating the dipole through $180°$ from the equilibrium position.

Ans. (i) 7.68×10^{-29} Cm, (ii) -3.07×10^{-23} J, (iii) 6.14×10^{-23} J.

92. An electric dipole of length 4 cm, when placed with its axis making an angle of $30°$ with a uniform electric field experiences a torque of 4 N m. Calculate the (i) magnitude of the electric field, (ii) potential energy of the dipole, if the dipole has charges of ±10 nC. Ans. (i) $2\times10^{10}NC^{-1}$, (ii) $-4\sqrt{3}J$

93. *A point-charge +Q is fixed at the origin of the coordinate system. A small electric dipole of dipole moment $\vec{p}$ pointing away from the origin along the X-axis is released from rest at a point **far away** from the origin. Find (i) the kinetic energy of the dipole when it reaches a distance d from the origin, and (ii) the force on the charge +Q due to the dipole at this moment.

Ans. F = $\frac{1}{4\pi\varepsilon_0}\frac{2pQ}{d^3}$, The force $\vec{F}$ acts along the X-axis.

94. *Two electric dipoles of moments p_1 and p_2 are in straight line. Show that the potential energy of each in the presence of the other is $-\frac{1}{4\pi\varepsilon_0}\frac{2p_1p_2}{r^4}$, and the interaction force between them is $-\frac{1}{4\pi\varepsilon_0}\frac{6p_1p_2}{r^4}$, where r is the distance between the dipoles (r is much greater than the length of the dipole).

Ans. $U=-p_2E_1=-p_2\times-\frac{1}{4\pi\varepsilon_0}\frac{2p_1}{r^3}=-\frac{1}{4\pi\varepsilon_0}\frac{2p_1p_2}{r^3}$, $F=-\frac{dU}{dr}=-\frac{d}{dr}\times-\left(\frac{1}{4\pi\varepsilon_0}\frac{2p_1p_2}{r^3}\right)=-\frac{1}{4\pi\varepsilon_0}\frac{6p_1p_2}{r^4}$.

CAPACITORS

SECTION A

1. Two isolated metallic solid sphere of radii R and 2R are charged such that both of these have same charge density σ. The spheres are located far away from each other and connected by a thin conducting wire. Find the new charge density on the bigger sphere. **Ans.** $\frac{5}{6}\sigma$.

2. A parallel-plate capacitor of plate area A = 600 cm^2 and plate separation d = 2.0 mm is connected to a D.C. source of 200 V. Calculate

(i)the magnitude of the uniform electric field $\vec{E}$ between the plates

(ii)the charge density σ on any one plate. **Ans.** $8.85\times10^{-7}\,Cm^{-2}$.

3. The area of the parallel plates of an air-capacitor is 0.20 m^2 and the distance between them is 0.01 m. The potential difference between plates is 3000 V. When a 0.01m thick sheet of an insulating material is placed between the plates, the potential difference decrease to 1000 V. Determine:

(i)capacitance of the capacitor before placing the sheet,

(ii) charge on each plate,

(iii) dielectric constant of the material,

(iv) capacitance of the capacitor after placing the dielectric and

(v) permittivity of the dielectric ε.

Ans. (i) $1.77\times10^{-10}F$, (ii) $5.31\times10^{-7}C$, (iii) 3.0, (iv) $5.31\times10^{-10}F$, (v) $2.65\times10^{-11}Fm^{-1}$.

4. *A parallel-plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength $10^7\,Vm^{-1}$. If the field is not to exceed 10% of the dielectric strength, find the minimum area of the plate required to have a capacitance of 50 pF.

Ans. 19 cm^2.

5. *A 1.0 μF capacitor C_1 and a 2.0 μF capacitor C_2 can separately withstand maximum voltages V_1 = 6.0 kV and V_2 = 4.0 kV respectively. What maximum voltage will be the system C_1 and C_2 withstand if they are connected in series? **Ans**.9.0 kV.

6. *A parallel-plate capacitor contains one mica sheet of thickness $d_1 = 1.0\times10^{-3}$ m and one fiber sheet of thickness $d_2 = 0.5\times10^{-3}$ m. The dielectric constant of mica and fiber are 8.0 and 2.5 respectively. Fiber breaks down in an electric field of $6.4\times10^{6}\,Vm^{-1}$. What maximum voltage can be applied to the capacitor? **Ans**. 5200 V.

7. (a)A 900-pF capacitor is charged by a 100-V battery. How much electrostatic energy is stored by the capacitor?

(b) The capacitor is disconnected from the battery and connected in parallel to another 900-pF capacitor. What is the energy stored by the system? **Ans. (a) 4.5×10^{-6} J, (b) 2.25×10^{-6} J.

8. The plates of a capacitor have an area of 90 cm^2 each and are separated by 2.5 mm.

(a)How much energy is stored in capacitor? Supply voltage is 100 volt.

(b) How much when it is filled with a dielectric medium of K = 3 and then charged ?

(c)If it is first charged as an air capacitor and then filled with the dielectric, then ?

9. *A 2 μF parallel-plate capacitor with a dielectric slab (K = 5) between the plates is charged to 100V and then isolated.

(a)What will be the p.d. if the dielectric be removed?

(b)How much wok would be done in removing the dielectric? **Ans**. 500 V, 0.20 J.

10.*The capacitance of a parallel-plate capacitor is 50 pF and the distance between the plates is 4 mm. It is charged to 200 V and the charging- battery is removed. Now a dielectric slab (K = 4) of thickness 2 mm is placed between the plates. Determine :

(i)final charge on each plate,

(ii)final potential difference between the plates,

(iii) final energy in the capacitor

(iv) energy-loss.

Ans. (i) the charge 10^{-8} C will remain as such. (ii) 125 V. (iii) 6.25×10^{-7} J. (iv) 3.75×10^{-7} J.

11. A parallel-plate capacitor of capacitance 100 μ F is charged to 200 V. after dis-connecting it from the battery, using an insulated handle the distance between the plate is doubled. Find

(i)potential difference between the plates, and

(ii) energy stored in the capacitor, after the separation between the plates has been increased.

Ans. (i) 400 V. (ii) 4 J.

12. *A 10 μ F capacitor is charged by a 30 V D.C. supply and then connected across an uncharged 50 μ F capacitor Calculate

(i) the final potential difference across the combination, and

(ii) the initial and the final energies. How will you account for the difference in energy ?

Ans. (i) 5.0 V. (ii) 4.5×10^{-3} J, 0.75×10^{-3} J.

13. *A parallel-plate capacitor has a plate area of 100 cm^2 and a plate separation of 2 cm. it has been charged up to 3000 V by a battery. Now (i) after disconnecting the battery, (ii) keeping the battery connected, the difference between the plates is increased to 5 cm. Find in each case the intensity of electric field between the plates and the energy of the capacitor.

Ans. (i) $1.5\times10^{5} Vm^{-1}$, 5×10^{-5} J. (ii) $6\times10^{4} Vm^{-1}$, 0.8×10^{-5} J.

14.*An electric field $E_0 = 3\times10^4 Vm^{-1}$ is established between the plates, 0.05m apart of a parallel-plate capacitor. After removing the charging battery an uncharged metal plate of thickness t = 0.01 m is inserted between the capacitor plates. Find the p.d. across the capacitor: (i) before, (ii) after the introduction of the metal plates. (iii) If a dielectric slab (K = 2) were introduced in place of metal plate.

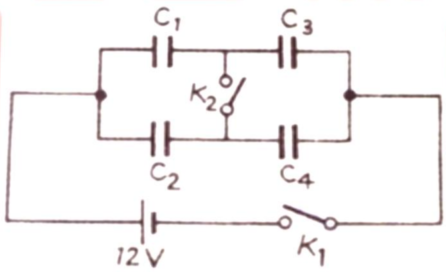
Ans. (i) 1500 V. (ii) 1200 V. (iii) 1350 V.

15.*A parallel-plate capacitor is charged to a certain potential difference. When a 3.0 mm thick slab is slipped between the capacitor plates then to maintain the same p.d. between the plates the plate separation is to be increased by 2.4 mm. Find the dielectric constant of the slab.

Ans. 5.

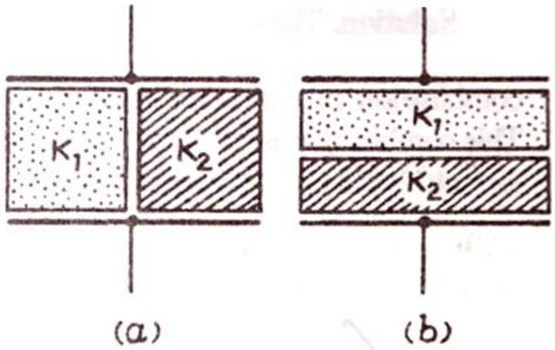
16.*In the given network, find the charge on each capacitor (i) when the key K_1 is closed and K_2 is open (ii) when the key K_1 and K_2 are closed. Take

$C_1 = 1\mu F, C_2 = 2\mu F, C_{13} = 3\mu F, C_4 = 4\mu F,$



Ans. (i) 9 μ C , 16 μ C. (ii) 8.4 μ C, 16.8 μ C, 10.8 μ C, 14.4 μ C.

17.A parallel-plate capacitor has plates each of are A and separation d. Two dielectrics of dielectric constants K_1 and K_2 are filled between the plates in two arrangements as shown. Find out the capacitance of the capacitor in each of the arrangements (a) and (b).

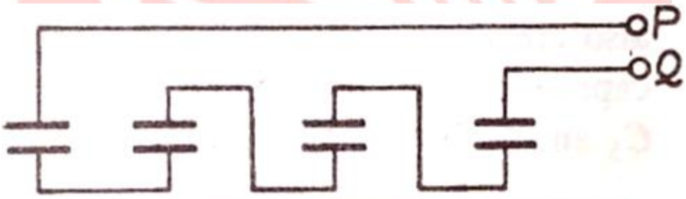


Ans. $\left(\frac{K_1K_2}{K_1+K_2}\right)\frac{2\varepsilon_0 A}{d}$

18.*A capacitor consists of 7 square plates of side 2.0 cm placed one above the other with 1.0 mm thick mica (k= 6) foils between them. The first, third, fifth and seventh plates are connected to one point and the second, fourth and sixth plates to another point. If the potential difference between these points be 300 V then calculate the stored energy in the capacitor.

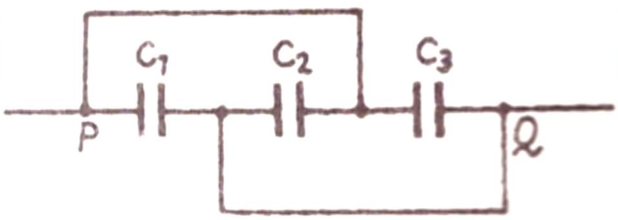
Ans. 5.71×10^{-6} J.

19. In the combination of four identical capacitors shown, the equivalent capacitance between points P and Q is 1 μF. Find the value of each separate capacitance.



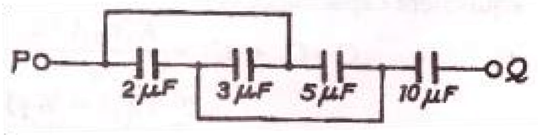
Ans. 4 μF.

20. Calculate equivalent capacitance between the points P and Q in the figure shown.



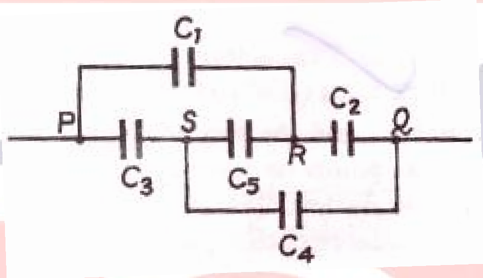
Ans. $C = C_1 + C_2 + C_3$.

21. Four capacitors are connected, as shown. Calculate the equivalent capacitance between the points P and Q.



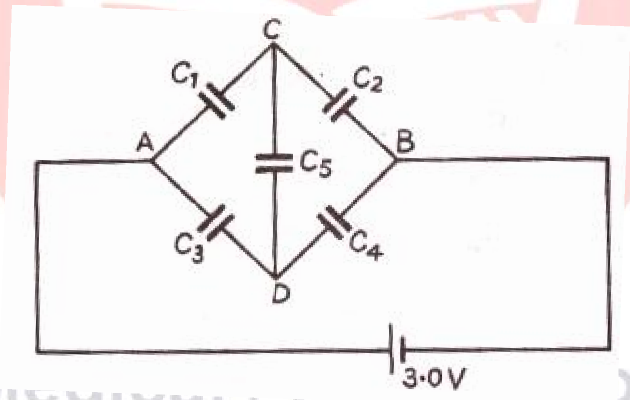
Ans. 5 μ F.

22. In the network of capacitors given below find the effective capacitance between the points P and Q. Given : $C_1 = C_2 = C_3 = C_4 = 4\mu F$ and $C_5 = 5\mu F$. If a 10 V battery be connected across P and Q what will be the charges on the capacitors ?



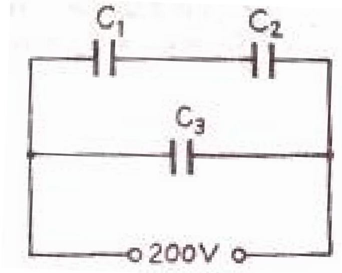
Ans. 4 μ F, 20 μ C. The charge on C_5 is zero.

23. Find the equivalent capacitance of the following circuit between the junctions A and B, given $C_1 = C_2 = C_3 = C_4 = 10\mu F$ and $C_5 = 5\mu F$.



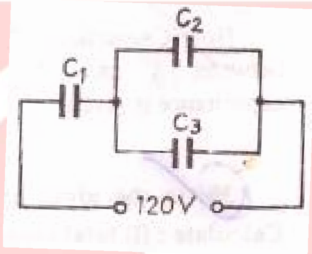
Ans. 10 μ F.

24. **In the given arrangement of capacitors, C_1 = 2.0 μ F, C_2 =6.0 μ F, C_3 =2.5 μ F. Calculate: (i) total capacitance, charge and energy of the system, (ii) charges on separate capacitors, and (iii) p.d.'s across separate capacitors.



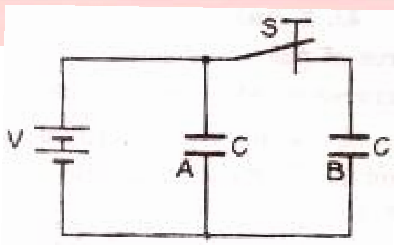
Ans. (i) 4.0 μ F, 800 μ C, 0.08J. (ii) 300 μ C, 500 μ C. (iii) 150 V, 50 V, 200 V.

25.*Three capacitors $C_1 = 15\,\mu$ F, C_2=25 μ F and C_3=35 μ F are connected to a 120 V supply as shown. Find (i) the equivalent capacitance of the system and energy stored in it (ii) charges and potential differences on C_1, C_2 and C_3.



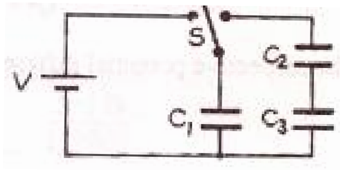
Ans. (i) 12 μ F, 8.64×10^{-2} J. (ii) 1.44×10^{-3} C, 96 V, 24 V, 600 μ C, 840 μ C.

26. In the given network, two identical parallel-plate capacitors are connected to a battery with the switch S closed. The switch is opened and the free space between the plates of the capacitors is filled with a dielectric K = 3. Find the ratio of the total electrostatic energy stored in both the capacitors before and after the introduction of the dielectric.



Ans. 0.6.

27. When the switch S in the figure is thrown to the left, the plates of the capacitors C_1 acquire a potential difference V. Initially, the capacitors C_2 and C_3 are uncharged. The switch is now thrown to the right. What are the final charges Q_1, Q_2, Q_3 on the corresponding capacitors?

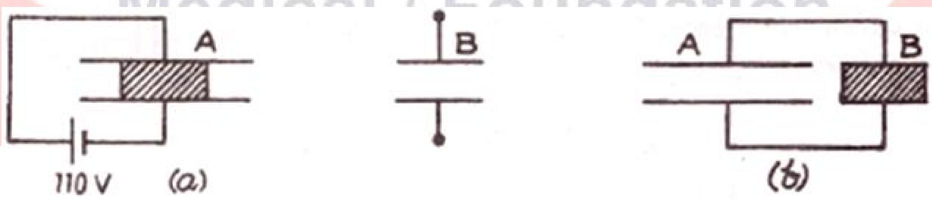


Ans. $\dfrac{C_1^2V(C_2+C_3)}{C_1C_2+C_2C_3+C_3C_1}, \dfrac{C_1C_2C_3V}{C_1C_2+C_2C_3+C_3C_1}$.

28. Two parallel-plate capacitors A and B have the same separation d = 8.85×10^{-4} m between the plates. The plate area of A and B are $0.04m^2$ and $0.02m^2$ respectively. A slab of dielectric constant (relative permittivity) K = 9 dimensions such that it can exactly fill the space between the plates of capacitor B. (i) The dielectric slab is placed inside A, as shown in figure (a). A is then charged to a potential difference of 110V. Calculate the capacitance of A and the energy stored in it.

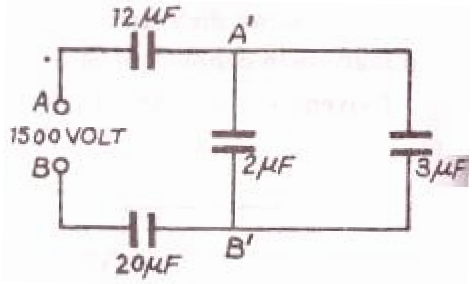
(ii) The battery is disconnected and then the dielectric slab is removed from A.

(iii) The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected, as shown in figure (b). Calculate the energy stored in the system. (Given: $\varepsilon_0 = 8.85\times10^{-12} C^2N^{-1}m^{-2}$)



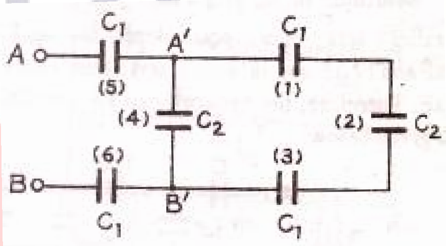
Ans. (i) 2×10^{-9} F, 1.21×10^{-5} J. (ii) 4.84×10^{-3} J. (iii) 1.1×10^{-5} J.

29. Find out in the figure given below, (i) equivalent capacitance between A and B, (ii) potential difference across the 2-μF capacitor, (iii) charge on the 3-μF capacitor and (iv) energy stored in the 20-μF capacitor.



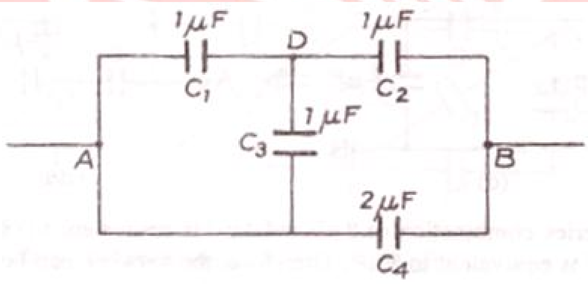
Ans. (i) 3 μ F. (ii) 900 V. (iii) 2700 μ C. (iv) 0.51 J.

30. If C_1 = 3 pF and C_2 = 2 pF, calculate the equivalent capacitance of the given network shown between point A and B.



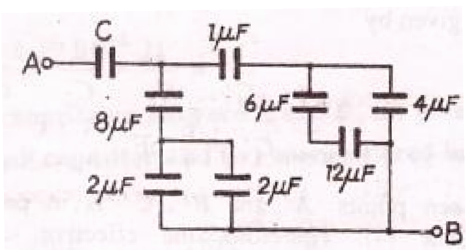
Ans. $\frac{60}{61}$ pF.

31. Determine the equivalent capacitance between A and B in the network shown below.



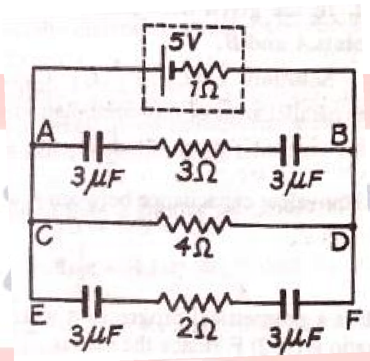
Ans. $\frac{8}{3}$ μ F.

32. In the given network of capacitors, the equivalent capacitance between the point A and B is 1 μ F. Find the value of the capacitors C.



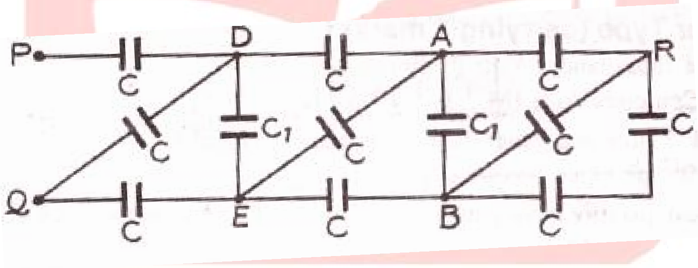
Ans. $\frac{32}{23}\mu$ F.

33. In the given circuit, calculate the charge on each capacitor in the steady state.



Ans. 6 μ C.

34. Find the equivalent capacitance in farad between the points P and Q, as shown in the figure below. C = 18F, C_1 = 12F.



Ans. 11F.

35. In the given network, each capacitor is of 1 F. Determine the equivalent capacitance between points A and B.

Ans. 2F

SECTION B

1. Assuming the earth as an insulated spherical conductor of radius 6400 km, calculate its capacitance. **Ans.** 711 μF.
2. 2. Each of the two metallic spheres of radii 15 cm and 10 cm has +100 μC of charge. They are connected by a wire. Find the common potential and final charge on each sphere. What is the amount of charge transferred through the wire? **Ans.** 80 μC. 20 μC charge is transferred from the smaller to the larger sphere.
3. Two insulated metallic spheres of capacitances 3.0 and 5.0 μF are charged to potentials of 300 and 500 volt respectively. They are connected by a wire. Calculate the common potential, charge on each sphere and the loss of energy.
   Ans. 425 V, 1.275×10^{-3} C, 2.125×10^{-3} C, 0.0375 J.
4. A parallel-plate capacitor with air between the plates has a capacitance of 8 pF. What will be the capacitance if the distance between the plates is reduced by half and the space between them is filled with a material of dielectric constant 6 ? **Ans.** 96 pF
5. What is the area of a plates of a 2-F parallel-plate capacitor with plate separation of 0.5 cm? Why do ordinary capacitors have capacitances of the order of microfarads?
   Ans. This is very large, unmanageable area. This is why ordinary capacitors of reasonable size have capacitances in microfarads. (However, electrolytic capacitors do have a much larger capacitances, $\approx$0.1 F, because of extremely small separation between the two plates.)
6. The area of each plate of a parallel-plate capacitor is 100 cm^2 and the intensity of electric field between the plates is 100 NC^{-1}. Find charge on each plate. $\varepsilon_0 = 8.85\times10^{-12} C^2N^{-1}m^{-2}$.
   Ans. $8.85\times10^{-12} C$.

7. An ebonite plate (K = 3), 6 mm thick, is introduced between the parallel plates of a capacitor of plate area $2\times10^{-2}m^2$ and plate separation 0.01 m. Find the capacitance.

Ans. 29.5 pF.

8. What should be the capacitance of a capacitor capable of storing 1 J of energy at 100 V D.C. supply ? **Ans.** 200 μ F.

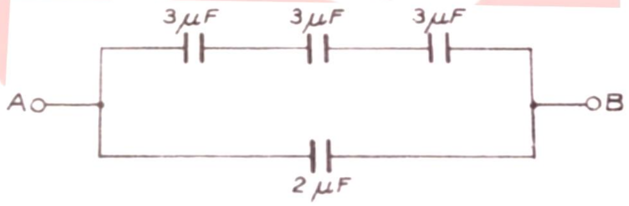
9. How would you combine 8, 12 and 24- μ F capacitors to obtain (i) minimum capacitance, (ii) maximum capacitance ? (iii) If a p.d. of 100 V be applied across the system, what would be the charges on the capacitors in each case ?

Ans. (i) 4 μ F. (ii) 44 μ F (iii) 400 μ C, 800 μ C, 1200 μ C, 2400 μ C.

10. Connect three capacitors of 3 μ F, 3 μ F and 6 μ F such that their equivalent capacitance is 5 μ F.

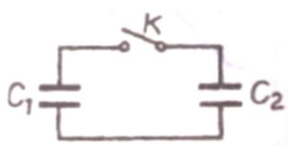
Ans. We shall connect 3 μ F and 6 μ F in series and the remaining 3 μ F in parallel of the series combination.

11. Find the equivalent capacitance between the points A and B of the given network of capacitors.



Ans. 3 μ F.

12. An 8- μ F capacitor C_1 is charged to a potential difference V_0 = 120 volt. The charging battery is then removed and the capacitor is connected in parallel to an uncharged 4- μ F capacitor C_2, as shown. (a) What will be the final p.d. across the combination ? (b) What will be the stored energy before and after the switch K is pressed ? What happens to the energy-difference ?



Ans. (a) 80 V. (b) 5.76×10^{-2} J, 3.84×10^{-2} J.

13. A battery of 10 V is connected to a capacitor of capacity 0.1 F. the battery is now removed and this capacitor is connected to a second charge capacitor. If the charge distributed equally on these two capacitors, find the total energy stored in the two capacitors. Further compare this energy with the initial energy stored in the first capacitor. **Ans.** 2.5 J, $\frac{1}{2}$.

14. Two capacitors, 25 μF and 100 μF, connected in series charged by a 120-V battery. The battery is then removed. The capacitors are now separated and connected in parallel. Find (i) p.d. across each, (ii) energy-loss in the process. **Ans.** (i) 38.4 V. (ii) 0.05184 J.

15. Three capacitors of 10, 15 and 30 μF are connected in series and on this combination a p.d. of 60 V is applied. Calculate the charge, potential difference and energy stored on each capacitor.
Ans. The capacitors are in series, the charge on each 3×10^{-4} C, 30 V, 20 V, 10 V, 4.5×10^{-3} J, 3.0×10^{-3} J, 1.5×10^{-3} J.

16. X and Y are two parallel-plate capacitors having the same area of plates and same separation between the plates. X has air between the plates and Y contains a dielectric medium of $\varepsilon_r = 5$. (i) Calculate the potential differences between the plates of X and Y. (ii) What is the ratio of electrostatic energy stored in X and Y? **Ans.** 10 volt, 2 volt. (ii) 5.

17. Find the capacitance of three parallel plates, each of area A meter^2 and separated by d_1 and d_2 meter. The in-between spaces are filled with dielectrics of relative permittivity ε_1 and ε_2. The permittivity of the free space is ε_0. **Ans.** $\frac{\varepsilon_1\varepsilon_2\varepsilon_0 A}{\varepsilon_2 d_1 + \varepsilon_1 d_2}$.